Weak Life Signal Detection Based on Wavelet Transform and Threshold De-noising Theory

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Abstract
When natural disasters and man-made disasters occur, people hope to search and rescue as soon as possible to the survivors. The radar echo signal is very weak and hard to extract in the life signal detection. In order to solve this problem, a new method based on wavelet transform and threshold de-noising theory is proposed. Through the studies of wavelet threshold de-noising method, the use of it in weak life signal de-noising in strong noise background, and the verification of simulation by Matlab. The results show that the wavelet threshold de-noising theory and method can effectively eliminate the noise signal from the weak life signal, which is an effective de-noising and extracting method, which is suitable for weak life signal detection. If this method is applied to the signal analysis of detection instruments, it should play an important role in the detection field of weak life signal.

Key words: Life Signal Detection, Wavelet Transform, Signal De-noising, Threshold Theory

1. INTRODUCTION
Life detection signal has the characteristics of signal weak, noise strong, frequency range low and stochastic strong. So it is one key factor to determine radar’s performance that the radar signal processing is good or bad for life detection radar. As the Doppler frequency shift of life signal is very weak, and the traditional Fourier transform signal processing has poor localization in the corresponding frequency domain as well as significantly localization in the time domain, it is difficult to locate the signal. Wavelet transform has good time-frequency localize properties in signal processing and characteristic of multi-resolution analysis, so it can extract transient information from non-stationary signal effectively and extract required life signal better (Chen and Huang, 2014; Lu and Yang, 2015).

2. WAVELET TRANSFORM THEORY
Wavelet transform is a tool to study the components of corresponding scale using the decomposition method after the data, function or operator is separated into different frequency components. In signal analysis, wavelet transform depends on two parameters: scale and time.

The main advantage of wavelet transformation is that it has a variable analysis window of time frequency. Its wide window can be used to analyze the low frequency signal and its narrow window can be used to analyze high frequency signal. Therefore, WT can provide the superior time frequency resolution for signal analysis within all the frequency scope (Stephane and Mallat, 2014).

Wavelet is the vibrating wave in a relatively short time interval, and the function used to represent wavelet is known as the wavelet function, noted as ψ(t), and ψ(t) ∈ L2(R)

Because of the need of application, Wavelet transform must meet the allowed condition

\[ C_\psi = \int_0^\infty \frac{\psi^*(\omega)}{\omega} d\omega < +\infty \] (1)

If \( f(t) \in L^2(R) \), and \( \psi(t) \) is allowed Wavelet, then

\[ (Wf)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \psi^\prime \left( \frac{t-b}{a} \right) dt, a > 0 \] (2)

It is the continuous Wavelet transform of \( f(t) \). In this equation, \( a \) is called scale factor or expansion factor, and \( b \) is called translation factor.

The formula of continuous wavelet transform is
The new formula of discrete wavelet transform is

$$W_{s}(a,b) = \int_{-\infty}^{\infty} x(t)\psi_{a,b}^{*}(t)dt = \langle x(t), \psi_{a,b}(t) \rangle$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t - b}{a}\right)$$

$$WT_{s}(a,b) = \sqrt{a} \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\Psi^{*}(a\omega)e^{j\omega b} d\omega$$

The new formula of discrete wavelet transform is

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k), j, k \in Z$$

According to the definition of Wavelet, we can see two characteristics of it. One is ‘small’, that is, there is compact support or similar compact support in time domain. The other is ‘volatility’, which means, DC component is zero and positive and negative is alternative (Ishihara and Kobayashi, 2016).

Because of these characteristics, for the small Doppler shift transformation in the actual life detection radar echo signal, it is better using an irregular wavelet function to approach sharply changing signal in a very short time than the sinusoid used in Fourier transform. The sampling frequency of response samples is 256Hz. Through the two divided-frequency function of WT, the particular frequency scope and rhythm corresponded by each dimension layer are illustrated as table 1.

**Table 1.** Frequency scope of each dimension layers in signal wavelet decomposition

<table>
<thead>
<tr>
<th>Dimensions layer</th>
<th>Frequency scope</th>
<th>Signal rhythm</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>64–128</td>
<td>high $\gamma$ wave</td>
</tr>
<tr>
<td>D2</td>
<td>32–64</td>
<td>$\gamma$ wave</td>
</tr>
<tr>
<td>D3</td>
<td>16–32</td>
<td>$\alpha$ wave</td>
</tr>
<tr>
<td>D4</td>
<td>8–16</td>
<td>$\beta$ wave</td>
</tr>
<tr>
<td>D5</td>
<td>4–8</td>
<td>$\theta$ wave</td>
</tr>
<tr>
<td>A5</td>
<td>0–4</td>
<td>$\delta$ wave</td>
</tr>
</tbody>
</table>

3. WAVELET THRESHOLD DE-NOISING IN LIFE SIGNAL DETACTION

Work principle of life detection radar is shown as figure 1. The signal receiving from radar’s antenna include the direct coupling wave between antennas, noted as $a(t)$, the reflection wave from medium interface, noted as $b(t)$, space clutter wave, noted as $r(t)$, and the signal of life parameters, noted as $s(t)$. So it is non-stationary random signal in strong noise background, and can be noted as

$$f(t) = s(t) + a(t) + b(t) + r(t)$$

![Figure 1. Work principle of life detection radar](image-url)
It is very difficult to extract the life parameters signal $s(t)$ from the echo signal $f(t)$ directly. At present, the common method to separate and extract signal is based on waveform analysis, high-frequency and DC component in radar echo signal can be removed by pre-treating after demodulating, then more standard life signal waveforms can be got by further digital filtering. As the non-contact life parameter signal is short-term, non-stationary, waveform and arrival signal unknown detected in strong noise background, adaptive filter and B-spline function fitting filter is selected for better effect, but their computation is complexity; FIR digital filter is simple, but phase distortion will appear; and signal processing method based on wavelet transform theory can solve the problem very well (Li and Lin, 2016; Yang and Wang, 2016).

3.1. Threshold De-noising Theory

One of measures to nonlinearly processing wavelet coefficients at present, named as the threshold de-noising arithmetic announced by Donoho, is mostly used. In this arithmetic, the selecting of the threshold function and the conforming of the threshold are the most basal problems. The major ways to conform threshold: soft threshold estimate based on Stein’s Unbiased Risk Estimate, fixed threshold, heuristics threshold and minimum maximum variance. The choice-of-way rule is shown in table 2. The energy of signal mainly concentrates in limited several coefficients in wavelet domain; however, the energy of noises is distributed in the whole wavelet domain. And the signal coefficient of wavelet transform is larger than the noise coefficient of wavelet transform. Therefore, a threshold can be set, when $W_{j,k}$ is under it, $W_{j,k}$ is mainly caused by noises but can be ignored, when $W_{j,k}$ is larger than it, $W_{j,k}$ is mainly caused by signals. Both soft and hard thresholds are used to process $W_{j,k}$ that is larger than the threshold.

<table>
<thead>
<tr>
<th>Selection of TPTR</th>
<th>Selection rules of threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘rigsure’</td>
<td>Unbiased Risk Estimate SURE of Stein for self-adaptive threshold selection</td>
</tr>
<tr>
<td>‘sqtwolog’</td>
<td>constant threshold selection, which equal to $\sqrt{2 \log \text{length}(s)}$</td>
</tr>
<tr>
<td>‘heursure’</td>
<td>heuristics threshold selection</td>
</tr>
<tr>
<td>‘minimaxi’</td>
<td>maximin theory threshold selection</td>
</tr>
</tbody>
</table>

The former leads to this part of wavelet coefficient close to zero according as a constant, the latter protects this part to stay directly. This method can be carried out by next three steps: (1) The decomposition of signal wavelets: Choose wavelet and decide the hierarchy of wavelet decomposition, then do N hierarchies wavelet packets decompositions for signals; (2) The quantification of threshold coefficients: Quantify high frequency wavelet coefficients by using a proper threshold, and achieve the estimation of wavelet coefficients; (3) The re-configuration of wavelets: To reconstruct wavelet according to the N hierarchy low frequency coefficients and high frequency coefficients quantified.

Hard threshold function

\[
\wedge W_{j,k} = \begin{cases} 
W_{j,k}, & |W_{j,k}| \geq \lambda \\
0, & |W_{j,k}| < \lambda 
\end{cases} \tag{8}
\]

Soft threshold function

\[
\wedge W_{j,k} = \begin{cases} 
\text{sgn}(w_{j,k})|w_{j,k}| - \lambda, & |w_{j,k}| \geq \lambda \\
0, & |w_{j,k}| < \lambda 
\end{cases} \tag{9}
\]

3.2. Threshold De-noising Method

The essence of signal processing based on wavelet theory is de-noising, and common methods of wavelet de-noising are modulus maxima de-noising method, threshold de-noising method, relevance de-noising method,
and shrinkage proportion de-noising method. In these methods, the threshold de-noising method is simple and effective.

The threshold de-noising method reconstruct the original signal by dealing with all level coefficients whose modulus is greater or smaller than a certain threshold value after wavelet transform separately and inverse transforming the Process wavelet coefficients. In the threshold de-noising, the threshold function reflects the different treatment strategy and different estimation methods to the mode of wavelet coefficients over or below the threshold. Hard threshold function and soft threshold function are common threshold function.

In hard threshold de-noising, the absolute value of the signal is compared with a specified threshold values, if the value is less than or equal to the threshold, let it be zero; else the value remains unchanged. In soft threshold denoising, the absolute value of the signal is compared with a specified threshold values, if the value is less than or equal to the, let it be zero; else it will be changed to be the value of that point minus the threshold. Generally, the signal dealt with hard threshold is rougher than soft-threshold, so using soft threshold de-noising to extract weak life signal that have low SNR (signal-to-noise ratio) can obtain better results (Vincent and Ajit, et al, 2015).

If \( \omega \) is the original wavelet coefficients, \( \eta(\omega) \) is the wavelet coefficients after threshold, and \( T \) is threshold, then

\[
I(x) = \begin{cases} 
1, & \text{when } x \text{ is true} \\
0, & \text{when } x \text{ is false} 
\end{cases}
\]  

(10)

represents indicating function, and the soft threshold function is

\[
\eta(\omega) = (\omega - \text{sgn}(\omega)T)I(|\omega| > T)
\]  

(11)

3.3. Threshold Estimation

Threshold estimation is an important factor in wavelet threshold de-noising method, if the threshold is too small, noise is still in the signal after de-noising; but if the threshold is too large, important signal characteristics will be filtered out, and deviation will be caused. From the intuitive view, for a given wavelet coefficient, the threshold is greater when the noise greater (Slotnick and Carney, 2015).

Common method of threshold is Visushrik threshold, SURE-Shrink threshold, GCV threshold value, etc. In these methods, the threshold in SURE-Shrink estimation is estimated under the SURE criteria, which is the unbiased estimation in the standard deviation criteria. The conclusion is specific to soft-threshold function, and the threshold value is near the ideal.

If the wavelet coefficients of original signal is estimated by the shrinkage of soft-threshold function, that is

\[
\hat{X}_i = \eta_i(Y_i) = (Y_i - \text{sgn}(Y_i)t)I(|Y_i| > t) \quad i = 1, 2, ..., N
\]  

(12)

Then the choice of threshold can be defined by the following risk function:

\[
R(t) = \frac{1}{N} \| \hat{f} - f \|^2
\]  

(13)

Because of the Orthogonally of wavelet transform, risk function can be written as

\[
R(t) = \frac{1}{N} \| \eta_i(Y) - X \|^2
\]  

(14)

\[
T(t) = \frac{1}{N} \| \eta_i(Y) - Y \|^2
\]  

(15)

Then

\[
ET(t) = \frac{1}{N} E \| \eta_i(Y) - Y \|^2 = ER(t) + \sigma^2 + \frac{2}{N} E(V, \eta_i(Y))
\]  

(16)

When \( V \) obeys Gauss distribution, the following equation can be setting up.
In this equation, $P(|Y| > t)$ obeys binomial distribution, and the probability can be similar to the frequency of $|Y| > t$, then the risk function can be expressed as follows:

$$ER(t) = ET(t) - \sigma_n^2 + \frac{2\sigma_n^2}{N} \sum_{i=1}^{N} I(|Y_i| > t)$$

$$= \frac{1}{N} \sum_{i=1}^{N} (|Y_i| \wedge t)^2 + \sigma_n^2 - \frac{2\sigma_n^2}{N} \sum_{i=1}^{N} I(|Y_i| > t)$$

Which $I$ indicates function indicator function, $\wedge$ means taking the small one in two number. Thus, the best choice of the threshold can be got by minimizing the risk function, that is,

$$t^* = \arg \min_{t>0} ER(t)$$

And the choice of best threshold may be in a limited scope, that is $t^* \in \{Y_1, Y_2, \cdots, Y_N\}$. In actual applications, more satisfied de-noising result can be achieved by using SURE-Shrink threshold de-noising method, which is a low-error method in threshold de-noising.

Because the SNR of signal received by life detection radar is very small, high-frequency coefficients generated in Wavelet transform, including life signal and noise signal, constitutes high frequency coefficient vector of the signal, the high frequency part in life signal will be removed as noise signal in general threshold selection method, but it will be reserved in SURE-Shrink threshold selection rules based on Stein Unbiased likelihood estimation.

4. MATLAB SIMULATION OF WAVELET DE-NOISING

Generally, the strong noise in detected weak life signal is mainly frequency interference signal, so we use the signal which amplitude is 1 and frequency is 0.7Hz to simulate human heart rate signal, use the signal which amplitude is 10 and frequency is 50Hz to simulate background noise signal, then the life signal in the strong noise background may be similar to the sum of these two signals and the noissin noise signal provided by Matlab. Though de-compositing signal by Db3 wavelet, estimating signal threshold by SURE-Shrink, and realizing by Heursure function, the simulation results show in figure 2.
b. Background noise simulation signal

c. Noisy simulation signal

d. De-noising signal

Figure 2. Simulation results of signal de-noising

From these results, we can found that wavelet threshold de-noising method can remove noise from weak life signal in strong noise background better, and a better life signal can be achieved.

5. CONCLUSIONS

Because of the characteristics of life signal, the traditional Fourier transform cannot be used in de-noising and signal extraction of it. But wavelet transform can analysis noise signal in time domain and frequency domain, it is suitable for transient signal detection, and can be an effective de-noising and extraction method for weak life signal. So it is very suitable for the detection of weak life signal, and can play an important role in the life detection area.

REFERENCES


