New Pneumatic Artificial Muscle Force Model Using Machine Vision
Volume Measurement and Virtual Work

Miha Pipan¹, Andrej Kos¹,Milovan Lazarević²,Niko Herakovič¹
¹Faculty of Mechanical Engineering, University of Ljubljana, Slovenia
²Faculty of Technical Sciences, University of Novi Sad, Serbia

Abstract There are a lot of different approaches for calculating forces generated with a Pneumatic Artificial Muscle (PAM). The majority of models that use virtual work are good approximations. Other methods are complex and need exact internal structure of the modeled muscle. In this paper we present a new method for obtaining PAM force module, that doesn’t need a PAM’s internal structure and gives better results in comparison to Chou’s most used virtual work model. With the use of acquired images of PAM’s in different pressure and contraction states we calculated a change of internal volume of the muscle and used it to make a new improved force model. The new model is compared to Chou’s model and experimental data. Also further improvement of the PAM force model is suggested.

Key words: pneumatic artificial muscle, virtual work, machine vision, force model.

1. INTRODUCTION
A Pneumatic Artificial Muscle (PAM) actuator is a pneumatic actuator that contracts when internal air pressure is increased. The pressure in PAM is generally controlled with pneumatic valves and with them the contraction of the muscle can be controlled. (Figure 1) (Festo, 2012).

![Figure 1. A PAM generating force and contraction with an increase of internal pressure P.](image)

The actuator’s main part is a rubber bladder that contains reinforcement fibers. The reinforcement fibers must be correctly oriented in respect to actuator’s main axis to act as a leverage to generate axial force. As the fibers change their orientation during contraction the generated force decreases. Also the spring and damping coefficients are not constant and therefore the control algorithms must have very fast control loops to achieve good dynamic characteristics. To achieve a fast control loop the pneumatic valves must have as fast response as possible and a good control algorithm. The advanced control algorithms for accurate and fast position control must be therefore developed from an accurate static and dynamic PAM model to achieve best possible results.

First step in making a new improved mathematical model of PAM’s generated force is to measure the volume in the different states of contraction under different pressure and load and then with the use of the virtual work theorem calculate the generated force.

2. MEASUREMENT OF VOLUME CHANGE OF PAM AND CALCULATION OF VIRTUAL WORK
The Chou and Hannaford model (Chou and Hannaford, 1996) is based on virtual work, where input work \( W_{in} \) in PAM is expressed as (Eq. 1). This equation is also our basis for developing a new improved model.

\[
dW_{in} = \int_{S_1} (P - P_0) d\ell_i d\mathbf{s}_1 = \hat{P} dV \tag{1}
\]

The \( P \) is absolute pressure in PAM, \( P_0 \) - ambient pressure, \( d\ell_i \) – inner surface displacement, \( d\mathbf{s}_1 \) – area vector, \( \hat{P} \) – relative pressure and \( dV \) - change in volume. Output work \( W_{out} \) is equal to the input work (Eq. 2) without any loss. \( dW_{out} \) – output work, \( F \) – generated force, \( dL \) – axial displacement. By combining the two equations we see that a generated PAM axial force \( F \) is a function of change of volume that depends on the axial displacement (Eq. 3).
\[ dW_{\text{out}} = -F \, dl \]  
\[ F = -\dot{p} \frac{dV}{dl} \]  
\[ (2) \]  
\[ (3) \]  

The Chou model then suggests, that the fiber length does not change and the generated force is calculated as a function of change in fiber angle (Eq. 4)

\[ F = \frac{\pi D_0^2}{4} (3 \cos^2 - 1) \]  
\[ (4) \]

This model does not take into account that fibers can stretch and therefore the volume changes. Some attempts were made to improve this model with including friction loss in a bladder (Tondu and Lopez, 2000; Tsagarakis and Caldwell, 2000). In this paper we try to improve this model with measuring the change of PAM volume in different pressure and contraction states and fit a polynomial function to this data. This polynomial will be a function with two variables: pressure \( P \) and contraction \( S \) (Eq. 5).

\[ V(P, S) = p_{n,0}P^n + p_{0,n}S^n + p_{n-1,1}P^{n-1}S + \cdots + p_{1,1}P S + p_{0,1} S + p_{1,0} P + p_{0,0} \]  
\[ (5) \]

The degree of the polynomial is determined in such a way, that the coefficient of determination \( R^2 \) is as high as possible and that the warping of function between measured points doesn’t occur. After all the polynomial coefficients are calculated, the generated force is calculated using the derivative of this function as shown in (Eq. 3).

3. EXPERIMENTAL SETUP

To measure the volume of the muscle and the generated force an experimental setup was constructed (Figure 2). The pneumatic muscle was mounted on low friction linear guides with positional mechanism. The contraction of the muscle during the experiments was fixed in different positions while the internal pressure was increased from \( P = 1 \) bar to \( P = 6 \) bar. During the incremental increase of pressure the images of PAM were taken. The internal pressure \( P \) in the PAM was controlled by two fast switching valves, the pressure sensor and an embedded PCX controller with a control algorithm (Figure 2).

![Figure 2](image-url)  
**Figure 2.** Experimental setup for the image acquisition of PAM’s volume. A – PC with Matlab, B – linear guides with clamping device, C – pressure sensor, D – displacement sensor, E – force sensor, F – fast switching valves, G – high resolution industrial camera and H – PCX controller

To achieve accurate volume measurements the camera had to be calibrated. We used the calibration algorithm for correction of radial and axial distortion due to camera lens deformation (Zhang, 1999). After the camera calibration the relative width of the camera pixels had to be measured. This was done with the use of the calibration checkerboard.
3.1 Experiment
For the experiment we chose the combination of pressures from 1 to 6 bar and contractions from 0 mm to maximum PAM’s contraction. This way we tested the muscle’s entire working area. The combinations of the measured parameters used in the experiment are shown in table 1.

The contraction of PAM was made in 2.5 mm steps from 0 to maximum contraction. Maximum contraction depends on the pressure in the muscle. At P = 6 bar without load the contraction increases to 25% of total PAM’s length.

Table 1. Combinations of the measured parameters

<table>
<thead>
<tr>
<th>Pressure [bar]</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contraction [mm]</td>
<td>0 -5</td>
<td>0 -23</td>
<td>0 -35</td>
<td>0 -40</td>
<td>0 -45</td>
<td>0 -50</td>
</tr>
</tbody>
</table>

4. CALCULATION OF PAM VOLUME
After acquisition of all PAM images, they had to be prepared for volume calculation. The entire analysis was done with the use of program package for numerical analysis – Matlab. This was done in 5 steps:
A) Define a region of interest and crop the picture to include only the PAM bladder.
B) Convert picture from RGB to binary.
C) Remove noise pixels and logo.
D) Invert picture for blob analysis.
E) Calculate the volume with blob analysis.

The first three steps are shown in figure 3.

After the image preparation was completed the volume of the pneumatic muscle was determined with the use of blob analysis in Matlab (C. González et al., 2004) and the use of Guldin’s rule (Figure 4) (Bojan et al., 2003) for calculation of volume for rotated area.
The volume is calculated as shown in (Eq. 6).

\[ V = 2\pi y_0 A_S \] (6)

We used the blob analysis in Matlab to calculate the \( y_0 \) and \( A_S \). The blob analysis and volume calculation was done in the following steps:

A) Load binary image.
B) Find blob.
C) Determine the center, minor axis, major axis and angle of the blob.
D) Rotate the blob and make two identical parts.
E) Make blob analysis of upper and lower parts, calculate their area and distance to rotation axis.
F) Calculate the average value of upper and lower area and center of gravity.
G) Calculate volume using Guldin’s rule.

The calculated volume presented the entire volume of the muscle. But for the calculation of virtual work only the inside volume of the air is needed. Therefore we subtracted the calculated volume with the volume of the PAM bladder. The bladder is made from rubber and is incompressible (Mott et al., 2008). The bladder volume can be therefore used as a constant. The volume was calculated by taking one PAM apart and by measuring the inner and outer diameter. The results of the measurement are presented in table 2.

<table>
<thead>
<tr>
<th>Length [mm]</th>
<th>Inner diameter [mm]</th>
<th>Bladder thickness [mm]</th>
<th>Volume ( V_p ) [mm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>20</td>
<td>2.05</td>
<td>28401</td>
</tr>
</tbody>
</table>

Figure 5 shows a cut part of PAM used for the measurement of dimensions and determination of the angle of fibers in the bladder.

Figure 5. Cut out part of PAM for the measurement of bladder dimensions (A) and the fiber angle \( \alpha \) (B).
We had to measure this angle to use Chou’s force model and compare it to ours. The reinforcement fibers are positioned at an angle $\alpha = 25^\circ$. After the calculation of the bladder volume we were able to calculate the volume of PAM in different states and compare it to the Chou force model.

5. NEW FORCE MODEL AND VALIDATION

A comparison between the experimental volume data and the Chou model (see Figure 6) shows that in general the Chou model corresponds with the experimental results of the volume change only at the PAM pressure of $P = 1$ bar. But at the PAM pressure of $P = 1$ bar the muscle can contract only approximately 3% of its length. Therefore at the PAM pressure of $P = 6$ bar, when the muscle contraction can reach full 25% of PAM’s length, the difference in the volume between the measured data and the Chou model is the greatest.

![Figure 6](image)

*Figure 6. The comparison of the Chou model and the gathered experimental data.*

The Chou model can therefore be improved with the simple function which takes into account the stretching of the bladder under different pressures and then use this new model to calculate generated force. With the use of MatlabCuve Fitting Toolbox we tested different degrees of Polynomial functions and concluded, that the best results are achieved with a third degree polynomial for pressure $P$ and Contraction $S$ (Eq. 7) in other words the third order surface polynomial function.

$$V(P,S) = p_{3,0}P^3 + p_{2,1}P^2S + p_{1,2}PS^2 + p_{0,2}S^2 + p_{1,1}P + p_{0,1}S$$  \hspace{1cm} (7)

All the coefficients were calculated and the coefficient of determination was $R^2 = 0.9978$. In figure 7 is the fitted polynomial surface function of the experimental volume data. The polynomial coefficients were stored in a Matlab matrix.

![Figure 7](image)

*Figure 7. Data fitted to experimental calculation of the PAM volume in different states.*
To calculate the generated force we had to use (Eq. 3) and (Eq. 5). The new force model was therefore calculated as the derivative of the function as shown in (Eq. 8). Our new force model is presented in (Eq. 9).

\[ F = -p \frac{dV(P,S)}{dS} \] (8)

\[ F = -\dot{P} \left( 3 p_0 S^2 + p_2 S^2 + 2 p_1 P S + p_{1,1} P + p_{0,1} \right) \] (9)

The new model was compared to Chou model and experimental data for evaluation. The difference between the two models and the experimental data is shown in figure 8. It can be seen that the new model is generally better than the Chou model on the entire scale for approximately 30%, but there is still a relative large difference between the new model and the experimental data.

![Figure 8](image.png)

Figure 8. A comparison of the new force model with the Chou model and the experimental data.

Since the change of PAM internal volume was accurately measured, the difference between the new model and also the Chou model must be in the work needed for the bladder deformation. But the accurate volume model and the virtual work did provide us with an improved model.

6. DISCUSSION AND CONCLUSION

The aim of the experimental research, presented in this paper was to measure the change of the PAM’s volume at different contractions and pressures and make a new force model based on virtual work. After the new model was made we compared it with the Chou model and the experimental data. The standard and the most used Chou model assumes that fibers in a PAM bladder don’t extend under pressure and therefore the volume doesn’t change. The fiber extension and volume change at different pressures also has influence on the Virtual Work model. In this paper we have shown that there is a change in the volume of the PAM in comparison with the Chou model and that the new force model is a better approximation of experimental data. The volume was measured with the use of high resolution machine vision camera for acquiring images of the pneumatic muscle at different states.

From the experimental results it is clear that the volume change is not only the function of the contraction but it depends also on the pressure change. The new force model and the improvements are clearly seen. As a result we could get more accurate generated force with the use of virtual work. But the improvement was approximately 30% so there is still room for improvement. To improve the model even further the model of losses in PAM bladder due to its deformation should be calculated and included in the force model. That would enable us to make an accurate model of forces generated by PAM.

REFERENCES