Adaptive Fractional Order Differential Image Enhancement Algorithm Based on Image Complexity

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Abstract
The paper provides adaptive fractional order differential image enhancement algorithm based on image complexity. In terms of the differential box counting principle of fractal image differential dimension, at first, the value of fractal dimension can be calculated. Then the mathematic relationship between fractional differential value and fractional order differential algorithm fractional order will be made. And last, based on the fractal dimension of image complexity, which will reflect the image complexity, the optimal fractional order will be determined, and the image complexity-based for adaptive fractional differential image enhancement will be implemented with the fractional differential algorithm. The results show that the algorithm will retain the image low frequency information, at the same time it will strengthen and improve the high frequency information; and it will effectively guarantee the best effect of the algorithm according to the optimal fractional differential algorithm fractional order determined by the adaptive image complexity.

Key Words: Adaption; Fractional Differential; Fractal Dimension; Image Complexity; Image Enhancement

1. INTRODUCTION
Fractional order differential mathematical theory has been established in seventeen century. Dr. Liouville and other mathematicians dedicated to perfect and develop the fractional order differential mathematical theory and built the basic system framework for it. So far, there is no unified time-domain definition expression. In many expressions, there are three main classic definitions: definition of Grünwald-Letnikow, definition of Riemann-Liouville and definition of Capotu (Miller, 1995; Riemann, 1876; Caputo, 1967). Till late nineteen century, fractional order differential theory has been put in to engineering. Mandelbrot discovered the self-similar phenomenon between the part and the whole; and made attempt to describe and analyze Brownian motion in fractal medium. From then on, many technicians and engineers have paid much more attention to the fractional order differential theory. In recent years, domestic scholars have found that fractional order differential have a broad prospect in application in underlying image processing. Till now the main applied achievements are: image enhancement, image edge detection, image segmentation, image novelty detection and etc.

The domestic main study achievements of fractional order differential theory in image processing are as follows: Pu Yifei and others introduced the fractional order differential into the image processing, and proved the elementary methods for the image processing with the fractional order differential. And they also took advantage of the frequency domain analysis to prove the fractional order differential will improve the information of image edge texture detail, while nonlinear retain the low frequency information (Pu, 2007; Pu, Zhou, Yuan, 2010). Yang Zhuzhong and others took advantage of the quality of weak derivative to test the edge of the image with weak noise signal, and avoided the effect of weak noise to precisely test the edge of the image (Yang, Zhou, Huang, 2007). Mathieu proposed the fractional order differential edge detection operator. He indicated when the operator was in \( 1 < v < 2 \), it was the time to test the edge. When it as in \( 1 < v < 1 \), the test would carry out, while the effect of noise would be avoided (Mathieu et al., 2003). Hereafter, Wang bin, Chen qingli and others proposed many improvement methods for fractional order differential templates. The general thinking is to make use of the relevance between the pixel and neighbor pixel, to make use of multi-scale structure attainable improved templates to improve the reinforcing effect of image edge texture information. The process would be implemented in the space domain. And these algorithms tested when the value of the fractional order is within 0-1, the fractional order differential image enhancement will be obtained. But the determination methods of optimal fractional order when processing the image have not provided (Wang, Yu, Lai, 2010; Wang, Pu, Zhou, 2012; Mathieu, et al., 2003; Zhang, Pu, Zhou, 2012).

Hence, how to determine the optimal fractional order has become the new research subject in the fractional order differential image processing. In recent years, Chen Dali and others proposed an adaptive fractional order differential algorithm based on the image statistic in order to determine the optimal fractional order (Chen, Chen, Xue, Fan, 2012). Wang Chengliang combined the characteristics of human eyes and proposed adaptive fractional order differential algorithm to determine the optimal fractional order (Wang, Lan, Zhou, 2011). Li Bo
proposed an adaptive fractional order differential algorithm based on the image pixel gradient in order to determine the optimal fractional order (Li, Xie, 2014).

The paper proposed adaptive fractional order differential image enhancement based on the image complexity to determine the optimal fractional order. Because the fractal dimension of the image fractal theory will reflect the image complexity, the paper has found the mapping relation between the fractal dimension and the fractional order. When processing the image, the value of the image fractal dimension will be calculated at first, then making use of the fractal dimension to determine the optimal fractional order, in order to realize the adaptive fractional order differential image enhancement of determined optimal fractional order based on the image complexity.

2. RELEVANT PRINCIPLE

2.1. Differential Dimension Box Calculation Based on The Image Complexity

Sarkar and Chaudhuri proposed the differential dimension box calculation (Sarkar, Chaudhuri, 1994). The basic thinking is as follows: divided M×M image into L×L sub block (1< L< M/2), and put the size of L×L×L' box onto the sub block. L' is the height of the sub block. To restrict differential dimension D between [2, 3], L' must meet the needs of [G/ L]=[M/L]. Among this, G stands for total number of gray levels. If in the (i, j)th check, the minimal value of gray level value vertically correspond to box Number C; while the maximal value of gray level vertically correspond to box Number D, then the box number dimension corresponding to the image of the check is n(i, j) = d - c + 1. The number of box covering the whole image is N = ∑n(i, j) the fractional dimension calculation formulation is

D = \frac{\log(N)}{\log(1/r)}, \quad r = \frac{L}{M},

if N is entered in, then value of fractional dimension D will be got; and D is the fraction distributed between [2, 3]. D displays the variation of the gray level value within the image area. The variation of the gray level value will reflect the complexity of image texture. The bigger of fractional dimension D is the more complexity of the image; vice versa.

2.2. Fractional Order Differential Image Enhancement Algorithms

(1) The differential expression is deduced by the definition of Grünwald-Letnikov fractional order differential. The definition of Grünwald-Letnikov fractional order differential is as follows (Miller, 1995):

\[ ^nD^s f(t) = \lim_{h \to 0} \frac{1}{h^n} \sum_{j=0}^{n} (-1)^j \frac{\Gamma(v+1)}{j!(v-j+1)} f(t-jh) \]

Gamama function is: \( \Gamma(n) = \int_0^{\infty} e^{-t^n} dt = (n-1)! \)

If one dimension function \( f(t) \) defines in the interval \([a, t]\), and parts it in terms of unit \( h = 1 \), to deduce \( n = \left[ \frac{t-a}{h} \right] = (t-a) \), then the differential expression of one dimension signal will be:

\[ \frac{d^n f(t)}{dt^n} \approx f(t) + (-v)f(t-1) + \frac{(-v)(-v+1)f(t-2)}{2} + . . . + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} f(t-n) \]

(2) The differentia mask deduced from differential expression

According to formulation (2) the coefficient of the differential template will be detruded. If the position coordinates of template Center is \( w(0,0) \), then the axial positive direction coordinates is \( w(1,0), w(2,0), w(3,0) \); And then x Axis negative direction coordinates is \( w(-1,0), w(-2,0), w(-3,0) \) . . . y Axis positive direction coordinates is \( w(0,1), w(0,2), w(0,3) \) . . . y Axis negative direction coordinates is \( w(0,-1), w(0,-2), w(0,-3) \). The coordinate value of the diagonal direction of the mask will be in turn and so on. According to formulation (2), the mask coefficients of eight directions (such as horizontal, vertical and diagonal) will be got as follows:
According to the principle of spatial filter image processing, the pending image pixels \( f(x,y) \) are located in the center of the mask \( w(0,0) \), the eight direction masks will be given to traverse the entire image, then the edge image \( \tilde{f}(x,y) \) can be got:

\[
\tilde{f}(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) \quad (3)
\]

Note: \( a = \frac{M-1}{2}, b = \frac{N-1}{2} \); \( w(s,t) \) is filter. Coefficient, \( f(x,y) \) is the image pixel value.

3. ADAPTIVE FRACTIONAL ORDER DIFFERENTIAL IMAGE ENHANCEMENT ALGORITHM

3.1. The Confirmation of Function Relationship Between Fractal Dimension And The Fractional Order Differential

According to the image differential dimension box calculation algorithm, the fractal dimension \( D \) will be calculated, which reflects the image complexity. As it is always the fraction distributed in \([2, 3]\), and the greater \( D \) value is, the more reflection of the image complex is; the smaller \( D \) value is, the smaller reflection of the image complexity is. The fractal dimension \( D \) reflects the image complexity.

In order to get the correspondence relationship between the fractional order and the fractal dimension of order, the method of \( V = D - 2 \) is adopted to confirm fractional order of fractional order differential algorithm. Because the values of the fractal dimension \( D \) is in \([2,3]\), \( V = D - 2 \) can guarantee \( V \) values of the fractional order in \([0,1]\), and fractional order \( V \) and the fractal dimension \( D \) is synchronous mapping relationship. That is to say, \( V \) increases with \( D \), and \( V \) decreases with \( D \). Therefore, the degree of change of the image complexity can be reflected from \( D \) to \( V \), thus the adaptive fractional order differential image enhancement algorithm can be achieved.

3.2. Algorithm Procedures

(1). According to the image differential dimension box calculation algorithm in fractal theory, fractal dimension \( D \) which represents the image complexity can be calculated. The greater the value of \( D \) is, the greater changes the image gray value is. It also reflects the complexity of image texture. And the smaller value \( D \) shows the smaller change of image gray value, and it also reflects the small image texture complexity;

(2). According to \( D \), determine the corresponding fractional order of fractional differential algorithm value \( D \) can be determined; because it is the fraction distributed in \([2, 3]\), the \( D - 2 \) calculation can be used to determine the numerical value \( V \) used in fractional differential, namely; This calculation \( V = D - 2 \) can guarantee the synchronous increasing and reducing relationship between \( V \) and \( D \).

(3). Making use of the fractional differential algorithm with the determined value, the differential operation of the image can be made in order to achieve adaptive enhancement of image target.
4. SIMULATION EXPERIMENT

4.1. Adaptive Fractional Differential Image Enhancement Edge Extraction

In order to verify the feasibility of the algorithm, three images will be arbitrary used for the validation of the algorithm. The fractional order of figure (a), (b), (c) is shown in table 1.

At first the differential dimension box of image can be calculated, through the calculating the fractal dimension values $D$ can be obtained, which reflected the image complex degree. Through the mapping relationship, the fractional order value $V$ will be received by $D$. Depending on the identified value of $V$, through the fractional differential algorithm, the enhanced image can be obtained. And edge details of the image are obtained through enhanced image minus the original image. Table 1 shows the fractal dimension value and fractional order value in terms of the calculation with differential box counting method. The order is the calculated values according to image complex degree.

<table>
<thead>
<tr>
<th>Image</th>
<th>$D$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure (a)</td>
<td>2.5455</td>
<td>0.5455</td>
</tr>
<tr>
<td>Figure (b)</td>
<td>2.4026</td>
<td>0.4026</td>
</tr>
<tr>
<td>Figure (c)</td>
<td>2.5058</td>
<td>0.5058</td>
</tr>
</tbody>
</table>

Different image complexity will get the different fractional order. Different fractional orders are used in the algorithms. Thanks to this, the realization of the Adaptive Fractional Differential enhancement can be achieved.

![Images](a_1) original image (a_2) Adaptive enhanced image (a_3) edge image

![Images](b_1) original image (b_2) Adaptive enhanced image (b_3) edge image

![Images](c_1) original image (c_2) Adaptive enhanced image (c_3) edge image

**Figure 1.** Adaptive Fractional enhancement of image

The picture (a_1) is a landscape picture. The picture (a_2) is an enhanced image after adaptive algorithm processing. The image retains the low-frequency information while enhancing the high-frequency edge details. We can see figure (a_2) is clearer than (a_1). The picture (a_3) is the details of the high-frequency edge information. And it is the enhanced high frequency information after the adaptive algorithm calculation. In order to verify the universality of an adaptive algorithm, respectively processing picture (b_1) and picture (c_1) with adaptive algorithm, we can get adaptive algorithm enhanced picture (b_2), picture (b_3), and adaptive algorithm for image
edge (c2), and picture (c3). The results show that the algorithm can according to the complexity of image, adaptively get the best fractional order for image enhancement, and achieve the optimal fractional order of image enhancement and extraction.

4.2. Comparison of Edge Detection Effect of Fractional Differential Algorithm and Adaptive Algorithm

In order to validate the superiority of the adaptive algorithm, the comparison will be made between the edge enhancement effect of the fractional order differential algorithm and the adaptive algorithm. The figure (b) - figure (j) is a fractional order differential enhancement. Fractional differential values were obtained from 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. As can be seen, when the numerical value is below 0.4, high-frequency detail information of the enhancement effect is not obvious, and the details of the information is not clear. With the fractional order increasing, the image detail information is gradually increased. But if the fractional order value is greater than 0.7, figure (I) – figure (J) shows that there is the pseudo edge exiting in the high frequency information. While the fractional order of fractional differential of adaptive algorithm based on the image complexity is 0.6125, the obtained edge image after calculation in this fractional value is not only clear, and also there is no pseudo edge. Therefore, the experimental results show that the adaptive fractional differential algorithm will not only determine the enhancement scale based on the image complexity, and it can get the optimal enhancement fractional order and the best image processing results, also has the feasibility.

![Figure 2. Contrast effect of image edge extraction](image-url)

5. CONCLUSIONS

This paper provides an adaptive fractional order differential image enhancement algorithm based on image complexity. The algorithm makes the use of the differential dimension box theory to calculate the fractal dimension which reflected the image complexity. Then the fractional order of differential image enhancement algorithm will be got on this figure. With this method, the optimal fractional order will be selected on the image complexity to achieve the enhancement optimal effect, in order to avoid the fake edge.
REFERENCES

Yang, Z Z, Zhou J L, Huang M(2007)“Fractional order differential was used to extract image edge”, Computer Engineering and Applications, 43(35), pp.15-18.