Structural Orientation Filter Based on Total Variation-L1 Model

Ning Yang*
School of Environment and Resource, Southwest University of Science and Technology, Mianyang 621010, China.
Corresponding author (E-mail: nyang@swust.edu.cn))

Yi Duan
Commonwealth Scientific and Industrial Research Organization (CSIRO), 1 Technology Court, Pullenvale, QLD 4069, Australian.

Abstract
To conquer the shortcoming of routine fracture-detection methods in the fact of noise, the paper introduces edge-preserving smoothing technique which is the SOF (structural direction filtering) based on the Total variation-L1 model. According to the seismic data is decomposed structure and texture components, including structural component is more clear and smooth area geometry, usually low-frequency signal; texture component signal and noise from seismic texture composed, respectively corresponding to high-frequency signal and a high frequency random noise. Finally, the SOF improves the quality of the seismic image.

Key words: Structural Orientation Filter, Seismic Image Enhance, Total Variation(TV), L1 Norm.

1. INTRODUCTION

Seismic image enhancement is different from the conventional seismic data noise-suppression technique. It aims to improve the interpretability of information in images, eliminate unnecessary information and provide superior input to further process images automatically. Seismic image enhancement can increase the dynamic ranges of selected features, and thus, considerably improve the accuracy of interpretation, analysis, recognition and measurement. This technique can also reduce multi-solutions for geophysical and geological interpretations.

High-density 3D seismic investigation requires precise exploration. In particular, high-resolution seismic images are required in researching small mass structures and in developing small fracture systems in clusters for the structural interpretation of the Sichuan Basin. Consequently, seismic image enhancement is crucial in processing seismic signals. Hocker (Hocker, 2002) proposed a fast seismic data interpretation technique based on seismic images, and then directly introduced the diffusion model into seismic image processing. Luo (2002) introduced the edge-preserving smoothing method for seismic data interpretation, and acceptable results were achieved. Marfurt (Marfurt, 2006) evaluated various filtering algorithms and determined that the details of seismic signals could not be enhanced without removing random noises; an image enhancement algorithm was then successfully applied to a variety of extracted seismic attributes.

In recent years, due to the development of the image denoising model and partial differential calculation, the quality of the image denoising method based on total variation theory (Paulsen,1996; Linh, 2008; Chen, 2010; Lu, 2015) of partial differential equations has been improved. The domestic researchers have made great progress in this field. Sun Xi-ping and Du Shi-tong et al.(Sun and Du,2004) application the coherent-enhancing anisotropic diffusion in seismic data processing, the results can only reflect the outline, details are not accurate. Chen Feng and Li Jinzong et al.(Chen and Li, 2004),Wang Xu-song (Wang, 2006) has used nonlinear anisotropic diffusion to enhance seismic image, the quality of processing results has obviously improved, but the edge region and the detail texture region is not distinguished. Yang Ning and He Zhen-hua et al. (Yang and He, 2010) proposed the seismic image enhancement algorithm based on the seismic complex channel information, but the method has not been derived from 3D seismic image enhancement algorithm. Yang Pei-jie (Yang, 2010) proposed the orientational edge preserving fault enhance, this technique improves the automatic recognition rate of the fault on the basis of maintaining orientation. Zhong Yong and Wang Shan-shan et al. (Zhong and Wang,2011) proposed the edge directed diffusion model, and discussed how to choose the edge orientation parameters. Based on directional filtering, Wen Xiao-tao and He Zhen-hua et al. (Wen and He, 2011) proposed geological body protruding display method, deduced and analyzed principles of selecting the parameters of the diffusion threshold, but the evaluation method has not discussed. Gao (Gao, 2011) analyzed the application of seismic texture in reservoir prediction, and points out the new seismic image enhancement method based on texture analysis.
In this paper, we present the seismic image enhancement based on the variational image decomposition model and partial differential equations (Meyer, 2002). Integrated with the structural orientation filter technology, this method extended the dynamic range of frequency domain and improved the spatial resolution of seismic image without any increase in the intrinsic information content of the data.

2. TOTAL VARIATIONAL SEISMIC DATA DECOMPOSITION MODEL

The 2D or 3D seismic data is expressed as:

\[ f = e + n \]  

where \( f \) is the original seismic image, \( e \) (original seismic data) is the structural component of the seismic data (i.e., the smooth part of the seismic image), \( n \) is not only noise but also the texture component of seismic data (the vibration of the seismic image). If the random noise and coherent noise in seismic data processing are effectively suppressed, \( n \) should be the real texture of seismic image. The seismic image decomposition is an inverse problem. It can be achieved using energy minimization and iterative total variation regularization. The general form is described by:

\[
\inf_{(e,n): e \in X_1, n \in X_2} \{ F(e) + \lambda F_2(n) : f = e + n \}
\]

where \( F_1(e), F_2(n) \geq 0 \) are functions and \( X_1 = \{ e : F_1(e) < \infty \} \), \( X_2 = \{ n : F_2(n) < \infty \} \) are spaces of functions or distributions. The constant \( \lambda > 0 \) is a tuning parameter. In our case, we recall the decomposition obtained by the ROF total variation(TV) minimization model for image denoising. Their functional is convex and therefore amenable to efficient minimization. We present in this paper an implementation of the TV-L\(^1\) model. So \( F_1(e) \) is expressed as

\[
F_1(e) = \text{TV}(e) = \int \left| \nabla e \right| \, dx dy dt
\]

Where \( \nabla e = \left( \frac{\partial e}{\partial x}, \frac{\partial e}{\partial y}, \frac{\partial e}{\partial t} \right) \).

The TV-L\(^1\) model is

\[
\inf_{(e,n): e \in BV(\Omega), n \in L(\Omega)} \{ \int _\Omega \left| \nabla e \right| + \lambda \left| n \right|_{L^1(\Omega)}, f = e + n \}
\]

TV minimisation is a convex variational method that plays an important role in imaging because it allows sharp discontinuities in a solution. Let \( X,Y \) be two finite-dimensional real vector spaces equipped with an inner product \( \langle \cdot, \cdot \rangle \) and a norm \( \| \cdot \| = \langle \cdot, \cdot \rangle ^{1/2} \). Let \( K : X \to Y \) be a continuous linear operator with an induced norm

\[
\| K \| = \max \{ \| Kx \| : x \in X \text{ with } \| x \| \leq 1 \} .
\]

We intend to solve the nonlinear primal problem, which yields the following equivalent primal–dual problem:

\[
\min_{x \in X} \max_{y \in Y} \langle Kx, y \rangle + G(x) - F^*(y).
\]

We suppose that \( F \) and \( G \) are “simple” functions because their proximal operators have a closed-form representation, as follows:

\[
\text{Pr}_{\alpha \gamma G}(x) = \arg\min_{z} \frac{1}{2} \| x - z \|^2 + \gamma G(z) - (\text{Id} + \alpha \gamma G)^{-1}(x),
\]

where the last equality comes from the definition of the sub-differential.

We consider a seismic data volume with size \( M \times N \times T : (e_{i,j,t})_{i\in\mathbb{N},j\in\mathbb{N},t\in\mathbb{N}} \). Let \( \mathcal{X} = \mathbb{R}^{MNT} \) be a finite dimensional vector space equipped with a standard scalar product, as follows:

\[
\langle e,n \rangle_{\mathcal{X}} = \sum_{i,j,t} e_{i,j,t} n_{i,j,t}.
\]

Gradient \( \nabla e \) belongs to the vector field \( \xi = \mathcal{X} \times \mathcal{X} \). For the discrete \( \nabla : \mathcal{X} \to \xi \), we use standard finite differences with Neumann boundary conditions, as follows:
\[(\nabla e)_{i,j,t} = \begin{pmatrix} (\nabla e)_{i,j,t}^1 \\ (\nabla e)_{i,j,t}^2 \\ (\nabla e)_{i,j,t}^3 \end{pmatrix}, \quad (9)\]

where

\[(\nabla e)_{i,j,t}^1 = \begin{cases} e_{i+1,j,t} - e_{i,j,t} & i < M \\ 0 & i = M \end{cases}, \quad (9)\]

\[(\nabla e)_{i,j,t}^2 = \begin{cases} e_{i,j+1,t} - e_{i,j,t} & j < N \\ 0 & j = N \end{cases}, \quad (9)\]

\[(\nabla e)_{i,j,t}^3 = \begin{cases} e_{i,j,t+1} - e_{i,j,t} & t < T \\ 0 & t = T \end{cases}. \quad (9)\]

We also introduce a scalar product in \(\xi\). For all \(p = (p^1, p^2, p^3)\) and \(q = (q^1, q^2, q^3)\),

\[\langle p, q \rangle_{\xi} = \sum_{i,j,t} p_{i,j,t}^1 q_{i,j,t}^1 + p_{i,j,t}^2 q_{i,j,t}^2 + p_{i,j,t}^3 q_{i,j,t}^3. \quad (10)\]

We define the discrete divergence operator \(\text{div} p : \xi \rightarrow \chi\) as the adjoint of the gradient operator \(\nabla = -\nabla^*\), which is defined through the following identity:

\[\langle e, \text{div} \rangle_{\xi} = -\langle p, \nabla e \rangle_{\xi}. \quad (11)\]

This identity is easy to check using the following definition:

\[(\text{div}(p))_{i,j,t} = \begin{cases} p_{i+1,j,t}^1 - p_{i,j,t}^1 & 1 < i < M \\ p_{i,j+1,t}^2 - p_{i,j,t}^2 & 1 < j < N \\ -p_{i-1,j,t}^3 + p_{i,j,t}^3 & 1 < t < T \\ -p_{i,j+1,t}^3 + p_{i,j,t}^3 & j = N \\ -p_{i,j,t+1}^3 + p_{i,j,t}^3 & t = T \end{cases}, \quad (12)\]

In a discrete setting, the TV-L1 model is read as

\[\min \lambda \|e - f\|_1 + \|\nabla e\|_1, \quad (13)\]

where \(f\) denotes the original image, and the solution for this problem \(e^*\) is the cartoon part. The discrete L1 norm is defined by \(\|e\|_1 = \sum_{i,j,t} |e_{i,j,t}|\) for a vector field \(\|\nabla e\|_1 = \sum_{i,j,t} \sqrt{((\nabla e)_{i,j,t}^1)^2 + ((\nabla e)_{i,j,t}^2)^2 + ((\nabla e)_{i,j,t}^3)^2}\).

We rewrite the problem in standard form as follows:

\[\min_{e} F \circ K(e) + G(e), \quad (14)\]

where \(G(e) = \lambda \|e - f\|_1\), \(F(e) = \|\nabla e\|_1\), and \(K = \nabla\). F and G are convex and simple functions, and thus, the conditions presented in Section 3.1 are satisfied. Therefore, we can derive the following primal–dual problem:

\[\min_{e} \max_{p} \{G(e) + \langle \nabla e, p \rangle - F^*(p)\}, \quad (15)\]

which we have used for its equivalence. Chambolle’s primal–dual algorithm solves the optimisation problem using an alternative minimisation scheme by applying two proximal operators, \(\text{Prox}_{\alpha F^*}\) and \(\text{Prox}_{\gamma G}\), at each step. If G is a “simple” function (its proximal operator), then we have the following fixed-point equation:

\[x^* = \arg \min_{x} F(x) + G(x) \Leftrightarrow x^* = \text{Prox}_{\gamma G}(x^* - \gamma F(x^*)) \quad (16)\]

Therefore, we have the following iteration

\[x_{k+1} = \text{Prox}_{\gamma G}(x_k - \gamma F(x_k)). \quad (17)\]
3. STRUCTURAL ORIENTATION FILTERING (SOF) BASED ON TV-L1 DECOMPOSITION MODEL

Analysing the two components of seismic data (e is the cartoon part, and n is the texture component with high-frequency random noise) and choosing which one is suitable to process with SOF seismic image enhancement will depend on the tendency diffusion factor according to the practical requirement (Yang et al., 2010).

Instantaneous frequency $\omega$ and instantaneous wave number $k$ are obtained through Hilbert transform. The instantaneous tendency is expressed as

$$p = k / \omega .$$

(18)

From the preceding formula, $e$ is described by

$$e = 1 - \frac{S - S_{mn}}{S_{mn} - S_{min}} \quad 0 \leq e \leq 1 .$$

Among these,

$$S_i = \sqrt{\sum_{m} \sum_{n} \left( \frac{p_{i,j} - \bar{p}}{nm-1} \right)^2} .$$

(19)

The structure tensor $T$ is constructed by the relationship between instantaneous tendency and dip angle, i.e.

$$T = \begin{pmatrix} \frac{1}{\sqrt{p^2+1}} & \frac{p}{\sqrt{p^2+1}} \\ \frac{p}{\sqrt{p^2+1}} & \frac{1}{\sqrt{p^2+1}} \end{pmatrix} .$$

(20)

The anisotropic diffusion operator is constructed by $e^*T$, as follows:

$$D = (e^*T) * K_\sigma ,$$

(21)

where $K_\sigma$ is a Gauss core with a low-pass filtering effect. The basic diffusion equation is derived from the anisotropic diffusion tensor $D$, as follows:

$$\frac{\partial u}{\partial t} = \text{div}(D \nabla u) ,$$

(22)

where $D$ is an anisotropic diffusion tensor; $\nabla$ is the gradient operator; and $u$ is the original seismic signal, which is substituted into the smoothing equation, as follows:

$$u(m+1) = u(m) + \frac{\partial u(m)}{\partial t} ,$$

(23)

where $u(m)$ is a primary seismic signal, $u(m+1)$ is the diffusion operator smoothed signal and $m$ is the number of iterations. If a high-frequency random noise is required to be removed, then only the texture component should be processed. In this case, $m$ ranges from 2 to 4. If an image needs to be enhanced, then both components should be processed. In this case, $m$ ranges from 5 to 20.

4. EXAMPLE

In Figure 1, the left side of Figure1.a is the post stack seismic data and the right side is the amplitude spectrum. Figure 1.b is the decomposed structure component (left) and amplitude (right). Structural component image is close to the original seismic image, and the statistical distribution of the amplitude spectra is also similar. By applying the parameters of the decomposition and the numerical calculation, the energy of the signal in range of 80-250Hz is slightly enhanced and the envelope curve is consistent with the amplitude spectrum envelope of the original data. Figure1.e shows the texture components (left) and amplitude (right). The dominant frequency of the texture component disagree with the original data and the energy is weaker than structural component. According to the amplitude spectrum, we can see that the energy of the signal in range of 80-250Hz is enhanced. It approximatively expands the frequency range. It indirectly improves the spatial
resolution of seismic images and highlights the seismic response characteristics of geological structures. As a result, it facilitates the study of seismic data interpretation.

Figure 1. Seismic data decomposition and amplitude spectrum. (a) Original seismic image and amplitude spectrum; (b) Structural component and amplitude spectrum; (c) Texture component and amplitude spectrum; (d) The red line is the original seismic image amplitude spectrum envelope. The blue line is the structure component for amplitude spectrum envelope. The green line indicates the texture component for amplitude spectrum envelope.
5. CONCLUSIONS

In this paper, the seismic amplitude attribute was combined with the TV-L1 seismic data decomposition theory to establish the mathematical model of the structure and texture components of seismic image. It proves that the texture information can be more accurately described by introducing the TV-L1 seismic data decomposition model and the SOF seismic image enhancement method based on the tendency of diffusion factor. According to the results, the seismic image enhancement method is not only approximate to the function of extension frequency, but also improves the spatial resolution of the seismic image and highlights the seismic response characteristics of the geological structure. It is beneficial to the study of seismic data interpretation. This method is different from conventional single seismic data statistical analysis or multidimensional mathematical transformation of seismic signal in wavelet base. It is a basic research work to measure the seismic response characteristics.

ACKNOWLEDGEMENTS

This work was supported by NSFC (41204068).

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