An Image Enhancement Method Based on Improved Fuzzy Set

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Abstract
At present, in related image processing field, the fuzzy set theory has also gained more successful applications to some extent. The image processing based on the fuzzy theory is an important research area. The information loss brought by 3D object projection in 2D image plane makes the image have the ambiguity exist in essence, thus there exists ambiguity when defining image characteristics such as the border, area and texture, therefore, because of the complexity of the image itself, uncertainty and inaccuracy or ambiguity may occur in the different stages of the processing. The algorithm in this paper improves the degree of membership function to make it suitable for the image fuzzy enhancement. The membership function curve is S-shaped, and at the inflection point, the function value on the left side drops rapidly while the function value on the right side rapidly rises. There is certain symmetry about the inflection point, and the function inflection point position can be decided by changing function parameters, and better enhancement effect can be gained by a small amount of iteration. By the experimental simulation, it is verified that the enhanced color image by adopting the method in this paper is bright in color on the visual effect with moderate increasing proportion of brightness and contrast, and best color maintaining performance and minimum chromaticity change, and also the image information entropy is improved when compared with the original image data.

Key words: Image Enhancement, Fuzzy Set, Membership Function.

1. INTRODUCTION
Image enhancement is the processing method to highlight certain information of an image according to specific needs, and at the same time weaken or remove some unwanted information, which is also a process to improve the image quality. Image enhancement processing does not increase the information of the original image, but can only enhance the recognition ability towards some sort of information, which makes the processed image more efficient than the original image in terms of some particular applications. The adoption of appropriate enhancement processing can process the original image that is blurred and even cannot be distinguished into the clear and distinct available image which is rich in a lot of useful information, so the image enhancement technique is widely used in many fields. The main idea of image enhancement based on fuzzy sets is the image enhancement processing method which combines the fuzzy mathematics theory and image enhancement, and the basic idea is to use the degree of membership function to transform the image to the fuzzy field from the spatial domain, so as to get the fuzzy feature plane on which the enhancement is processed towards the image, and finally the enhanced image is gained by transforming back to the spatial domain(Patrick and Nalin et al, 2015; Tamalika, 2014).

Image enhancement method, starting from the action domain of enhancement, can be divided into spatial domain enhancement and frequency domain enhancement. Spatial domain enhancement method directly processes the pixels in the image, and the spatial domain method processes the point operation on the image, and it can allow users to change the grey value of pixels in the image, so as to create a new image through point operation processing. Fundamentally, the spatial domain method bases on the mapping transformation of image, and the mapping transformation type used depends on the purpose of the enhancement. The frequency domain enhancement method deals with the transformation coefficient in a certain frequency domain of the image, and then the enhanced image is gained by the inverse transformation(Jorge and Alejandro, 2014). At present, many new enhancement algorithms make full use of surrounding neighborhood (the important information) to form a lot of local processing adjustment algorithm which mainly uses the statistical characteristic of the neighborhood. The image enhanced by the traditional histogram enhancement algorithm has the disadvantage of image detail information loss and noise amplification. In recent years, the fuzzy set theory has been widely in image processing. Generally speaking, the image, in the mapping process from 3D object to 2D image, inevitably has
the loss of information, so the images are born with fuzziness. The person’s vision for the image color level is
fuzzy and difficult to accurately distinguish. This leads to the definition of image edge, area and texture, and the
ambiguity in interpretation of the underlying image processing results. Therefore, the use of fuzzy information
processing technique in image processing has its inherent rationality and inevitability (Raju and Madhu, 2014;
Om Prakash and Puneet, 2012).

This paper first explains the principle of image enhancement and the traditional image enhancement
methods, then analyzes the fuzzy set theory and its degree of membership function. Based on the above research,
this paper puts forward an image enhancement method with improved fuzzy degree of membership function. By
the end of the paper is the experimental simulation and analysis, and the result proves the feasibility and
effectiveness of the algorithm in this paper.

2. IMAGE ENHANCEMENT

Image enhancement is the processing method to highlight certain information in an image according to
specific requirements and weaken or remove some unnecessary information. Meanwhile, it is also the process to
improve image quality. Traditional image enhancement algorithms are usually based on the statistic of the entire
image when determining the transform function, i.e. histogram equalization. In this way, the impact of the
details of some local regions is usually ignored in calculating the transformation of the entire image because it is
small in value; therefore, the enhancement effects of the local regions are usually not very ideal (Sana and

2.1. Linear Transformation

Piecewise linear transformation can be used in order to stress the gray-level space where the interesting
object is located and suppress the uninteresting gray-level spaces. Assume that the original image is \( f(x,y) \) and
the gray-level range of interesting objects is \([a, b]\), to extend the gray-level range to \([c, d]\), the corresponding
linear transformation formula \( g(x,y) \) is as follows.

\[
g(x,y) = \frac{d-c}{b-a} \times [f(x,y) - a] + c
\]

If the gray levels of most pixels are distributed within the range of \([a, b]\), \( M_f \) is the maximum gray level
of the original image and only a very small number of gray levels have exceeded this range, in order to improve
the enhancement effect, make

\[
g(x,y) = \begin{cases} 
\frac{(c/a)f(x,y)}{} & \text{if } 0 \leq f(x,y) < a \\
((d-c)/(b-a))[f(x,y) - a] + c & \text{if } a \leq f(x,y) < b \\
[(M_g - d)/(M_f - b)][f(x,y) - b] + d & \text{if } b \leq f(x,y) \leq M_f
\end{cases}
\]

This kind of linear transformation forces the pixels, the gray level of which is smaller than \( a \) and bigger
than \( b \) to transform into \( c \) and \( d \) and it enhances the layering of the gray levels of most pixels of the image, as
indicated in Fig.1.

(a) Original Image            (b) Transformation Result

Figure 1. Linear transformation of image

2.2. Non-linear Transformation

Non-linear transformation performs gray-level transformation on the image with non-linear transformation
functions, including logarithmic transformation and exponential transformation.

(1) Logarithmic Transformation
It refers to the logarithmic relationship between the gray level of the pixel point of the output image and that of the input image with a formula as follows.

\[ g(x, y) = \ln[f(x, y)] \]  

(3)

In order to expand the dynamic range of the transformation, some modulation parameters can be increased to the above formula. Then, the transformation formula will be as follows.

\[ g(x, y) = a + \frac{\ln[f(x, y) + 1]}{b \cdot \ln c} \]  

(4)

Here, \( a, b \) and \( c \) are the parameters introduced to adjust the position and shape of the curve. \( a \) is the intercept of Y-axis and it determines the transformation relation of the initial position of the transformation curve. \( b \) and \( c \) determine the change rate of the curve. When stretching the low gray-level region of the image while compressing the high gray-level region, this kind of transformation can be used and it can match the gray-level distribution of the image with the visual features of human.

2.3 Histogram Equalization Algorithm

The frequency of some image is quite large in low gray-level region, making it impossible to see the details of the darker regions in the image. In this way, the gray-level range of the image can be separated with histogram equalization and increase the gray level with small gray-level frequency. By adjusting the dynamic range of the gray levels, automatically increase the contrast of the entire image and make the details clear.

Assume that \( r \) represents the gray levels of the pixels of the image. Perform normalization processing on the gray levels, then \( 0 \leq r \leq 1 \). When \( r = 0 \), it is black; when \( r = 0 \), it is white. For a given image, the gray level of every pixel is random at the range of \([0,1]\). Use probability density function \( p_r(r) \) to represent the distribution of the gray levels of the image. It can be proven that to take the cumulative distribution function of \( r \) as the transformation function can produce an image with the gray levels distributed in uniform probability density. The result expands the dynamic range of the pixel value. In favor of digital image processing, discrete form is introduced. In the discrete form, use \( r^i \) and \( P_r(r^i) \) to represent discrete gray level and \( p_r(r) \) respectively and

\[ P_{r^i}(r^i) = \frac{nk}{n} \]  

(6)

Here, \( 0 \leq r^i \leq 1 \), \( k = 0, 1, 2, ..., n-1 \). \( r^i \) in the formula is the number of pixels with the gray level of \( r^i \) in the image. \( n \) is the total number of the image pixels and \( \frac{nk}{n} \) is the frequency in probability theory. The histogram equalization function formula of the image is as follows.

\[ S_k = T(r_k) = \sum_{j=0}^{i} \frac{n}{n} j \quad k = 0, 1, 2, ..., l-1 \]  

(7)

Here, \( k \) is the number of gray levels and the corresponding inverse transformation is \( r^i = T^{-1}(S_k) \).

Transformation function \( T \) shall meet the following conditions: a. Within the range of \( 0 \leq r \leq 1 \), the individual of \( T(r) \) increases monotonically, ensuring that the gray levels of the image remain the same order from white and black; b. For \( 0 \leq r \leq 1 \), there is \( 0 \leq T(r) \leq 1 \), ensuring that the gray levels after affine transformation remain within the allowable range. The reversibility of transformation function selectively tries
to highlight the interesting formation for human or robot analysis and restrain some useless information through enhancement processing in order to enhance the value of the image.

The histogram equalization transformation function is shown in Fig. 2. Assume that \( r \) and \( s \) represents the gray levels of the original image and enhanced image respectively. For simplicity, assume that the gray levels of all pixels have been normalized. When \( r = s = 0 \), the gray level of the pixel is black; when \( r = s = 1 \), it is white and when \( r \) and \( s \) are among \([0,1]\), it means that the gray level changes between black and white. The gray-level transformation function is \( s = T(r) \).

![Figure 2. Histogram equalization transformation function](image)

Histogram reflects all the gray levels of the image and performs transformation on the distribution of gray levels. Actually, since histogram is an approximate probability density function, to use discrete gray level as the transformation can barely obtain completely flat result. Besides, after the transformation, gray level is usually reduced. Histogram equalization processing is the histogram modification method based on cumulative distribution function transformation method. If the pixels of an image cover all possible gray levels and are distributed evenly, the image will have a high contrast and a versatile tone (P. Balasubramaniam and V.P. Ananthi, 2014; Wenjuan and Haibo, 2016). Fig. 3 is the comparison of the original image and the histogram of the equalized image.

![Figure 3. Comparison of the original image and the histogram of the equalized image](image)
By comparison, we can find that the original gray level is mainly distributed in the middle and low gray level, the number of pixels in the high gray level image is very small. After the histogram equalization processing, the image pixels in the high school low gray level distribution is more uniform, the image contrast is greatly improved.

2.4 Contrast Enhancement

Some image has low contrast, making the entire image unclear. At this time, revise the gray level of every pixel of the image according to certain rules so as to change the dynamic range of the gray levels of the image. Contrast enhancement revises the gray level of every pixel of the input image point by point according to certain rule in order to change the dynamic range of the gray levels of the image. it can extend the dynamic range of the gray levels and compress the range or compress in certain dynamic region and extend in other regions. Proper selection of gray-level transformation rule can increase the gray-level contrast of the interesting regions. Assume that the input image is \( f(x, y) \) and the processed image is \( g(x, y) \), then the contrast enhancement can be represented with the formula below.

\[
g(x, y) = T[f(x, y)]
\]  

Here, \( T \) refers to the gray-level mapping relationship of the points in the input image and the output image. Contrast enhancement method is indicated as Fig.4.

![Figure 4. Contrast enhancement](image)

For the low-contrast image with a histogram lower than 35 or higher than 210, if the image data is mapped to the entire gray-level range, the image contrast will be greatly increased (Ananthi and Balasubramaniam et al., 2012; Jie and Hui, 2016).

3. IMAGE ENHANCEMENT METHOD BASED ON IMPROVED FUZZY SET

3.1 Fuzzy Set and Its Membership Function

Assume that \( A \) is a fuzzy subset of domain \( U \), to any \( x \in U \), for a given mapping \( \mu_x \) from \( U \) to \([0, 1]\), namely \( \mu_x : U \to [0, 1] \ x \mapsto \mu_x(x) \). In other words, for \( x \in U \), if \( \mu_x(x) = 1 \), \( x \) is considered to belong to \( A \) completely; if \( \mu_x(x) = 0 \), \( x \) is considered not to belong to \( A \) completely. If \( 0 < \mu_x(x) < 1 \), \( x \) is considered to belong to \( A \) in the extent of \( \mu_x(x) \). Therefore, \( \mu_x(x) \) is called the membership of \( x \) to \( A \) and \( \mu_x(x) \) is called the membership function of the fuzzy set \( A \).

The fuzzy set \( A \) in the domain \( U \) is usually marked as \( A = \{ \mu_x(u)/u \} \). Fuzzy set can be applied to the circumstances whether the element belongs to or doesn’t belong to a certain set. Therefore, the membership of certain element to certain set can be considered. The membership varies within the range of 0 and 1. If \( \mu_x(u) \) is closed to 1, \( x \) highly belongs to \( A \) and if \( \mu_x(u) \) is closed to 0, \( x \) slightly belongs to \( A \). One membership function only determines a fuzzy set and a fuzzy set corresponds to a membership function. As fuzzy set is quite abstract, membership function will be frequently used to represent the fuzzy set afterwards (Ballman and Begeng, 1977; Celle, 1969).
3.2 Procedures of Algorithm of This Paper

The basic idea of the algorithm of this paper is to transform the image from the spatial domain to the fuzzy domain with membership function, obtain the fuzzy characteristic plane, perform enhancement processing on the image in the fuzzy characteristic plane and obtain the enhanced image after transforming it back to the spatial domain.

First, let’s assume that \( I \) represents an image with a size of \( M \times N \) and \( L \) color gradients and \( x_{mn} \) represents the gray-level value of point \((i, j)\) in the image \( I \). Define \( \text{mem}_t(x_{ij}) \) to represent the membership value of certain property of this point; in other words, we have defined the fuzzy sub-set mapping from the image \( I \) to the range of \([0,1]\). To be professional, we can have

\[
I = \{x_{ij}, \text{mem}_t(x_{ij})\}, \quad 0 \leq \text{mem}_t(x_{ij}) \leq 1, \quad i = 0,1,...M - 1, \quad j = 0,1,...N - 1 \tag{9}
\]

Image fuzziness mainly includes the following three steps: extraction of image fuzzy feature, modification of membership function value and inverse transformation of fuzzy domain.

Assume that \( h(g) \) refers to the number of pixels with a level of \( g \) in the image and for a given \( t \), the average values \( \text{mem}_b \) and \( \text{mem}_f \) of background and foreground color gradients can be represented with the following formulas respectively.

\[
\text{mem}_b = \frac{\sum_{g=0}^{L} gh(g)}{\sum_{g=0}^{L} h(g)}, \quad \text{mem}_f = \frac{\sum_{g=t+1}^{L} gh(g)}{\sum_{g=t+2}^{L} h(g)} \tag{10}
\]

The above \( \text{mem}_b \) and \( \text{mem}_f \) can be seen as the object values of the corresponding foreground and background to the given \( t \) and the relationship between a certain point in the image \( I \) and the described region shall be relevant to the difference between the color gradient of this point and the object value of the region it belongs to intuitively. Therefore, we have proposed the following membership definition function for point \((i, j)\).

\[
\text{mem}_t(x_{ij}) = \begin{cases} 
1 & \text{if } x_{ij} \leq t \\
\frac{1}{1+|x_{ij} - \text{mem}_t|/e} & \text{if } x_{ij} > t 
\end{cases} \tag{11}
\]

Here, \( e \) is a constant, which makes \( 0.5 \leq \text{mem}_t(x_{ij}) \leq 1 \). Transform the image from the level space of the spatial domain to the fuzzy characteristic plane (membership plane), modify the membership value with proper fuzzy technology and transform the data from fuzzy characteristic plane to the spatial domain of the image through fuzzy domain inverse transformation. The only way to transform the digital image to be enhanced to the fuzzy spatial domain is to determine the fuzzy membership function and then transform the digital image into the fuzzy domain to be defined as follows.

\[
\mu_{ij} = \cos \left( \frac{\pi}{2} \times \frac{x_{ij} - x_{\min}}{x_{\max} - x_{\min}} \right) \tag{12}
\]

Here, \( \mu_{ij} \) refers to the membership of the gray level \( x_{ij} \) of \((i, j)\) relative to the maximum level \( x_{\max} \), \( x_{\min} \) can be either 0 or the minimum level of the image as required and \( k \) is the adjustment parameter. Therefore, it can be seen from the membership function formula that the value range of \( \mu_{ij} \) after the digital image is transformed into the fuzzy domain is \([0,1]\). Map the image back to the spatial domain according to the formula below and get the final enhanced image.

\[
x_{ij}' = x_{\min} + (x_{\max} - x_{\min}) \cdot \arccos \left( \frac{1}{\mu_{ij}^2} \right) \cdot \frac{2}{\pi} \tag{13}
\]

In the entire enhancement process, the selection of parameters is of significant importance. The degree of membership of the function can be changed by adjusting parameters \( k \) and \( x_{\min} \). The information entropy of
the fuzzy set reflects the average image information and it is a statistical form of image feature. The following formula is for the calculating the information entropy of a given image sample set \( I \).

\[
H = \text{Entropy}(I) = \sum_{i=1}^{n} -p_i \log_2 p_i
\]

Here, \( H \) is the information entropy of the image. According to the entropy theory, the bigger \( H \) is, the more profound information the image contains. \( n \) refers to the classification number of sample set \( I \), \( p_i \) refers to the probability of the emergence of class \( i \) elements in \( I \). The greater the information entropy indicates that the \( I \) classification of the sample set is more dispersed, the smaller the entropy, the more concentrated the \( I \) classification of the sample set. When the probability of \( n \) classification is as large as \( \frac{1}{n} \), the information entropy achieves the maximum value, when \( I \) has only one class, the minimum value of the information entropy is 0.

4. EXPERIMENTAL SIMULATION AND ANALYSIS

To objectively evaluate color image enhancement effect by adopting this methods, this paper adopts the method based on the mean and variance measurement of the image, to a certain extent, respectively measures the contrast and brightness change index of color image enhanced by the algorithm when compared with the original input color image. The mean and variance measurement method of image are as follows:

\[
\Delta C = \frac{V(\lambda_{\text{out}}) - V(\lambda_{\text{in}})}{V(\lambda_{\text{in}})}
\]

\[
\Delta B = \frac{M(\lambda_{\text{out}}) - M(\lambda_{\text{in}})}{M(\lambda_{\text{in}})}
\]

In the above formulas, \( \lambda_{\text{in}} \) and \( \lambda_{\text{out}} \) are respectively the brightness component \( \lambda \) of input and output color image RGB space, \( V \) and \( M \) are respectively the functions to demand the variance and mean, \( \Delta C \) is the change proportion of the enhanced image contrast, \( \Delta B \) is the change proportion of the brightness, and when the value is positive, the contrast and brightness are enhanced, and the contrast and brightness decrease when the value is negative.

Meanwhile, in order to evaluate the color maintaining performance of the algorithm, the computation formula modeled on the above formula to define the chromaticity change ratio is as follows:

\[
\Delta H = \text{abs} \left( \frac{M(H_{\text{out}}) - M(H_{\text{in}})}{M(H_{\text{in}})} \right)
\]

Here, \( H \) is the information entropy of the image.

According to the above algorithm, 2 standard test images are used to realize the simulation experiment by using the Matlab software. The simulation test environment is Intel Celeon G1610 2.6GHz CPU and 1G SD memory hardware environment and MATLAB 2010 software environment. Comparison includes the overall visual effect of the image and image information entropy and other parameters. The experimental results are shown in Figure 5, 6 and Table 1, 2.

(a)Original image  (b) RGB color histogram
Figure 5. The experimental results of football image

Figure 6. The experimental results of onion image

Table 1 Experimental data of football image

<table>
<thead>
<tr>
<th>Enhancement method</th>
<th>Objective parameters</th>
<th>Contrast change (ΔC)</th>
<th>Brightness change (ΔB)</th>
<th>Chromaticity change (ΔH)</th>
<th>Image information entropy (H)</th>
<th>Algorithm efficiency (s)</th>
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Table 2 Experimental data of onion image

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<th>Brightness change (ΔB)</th>
<th>Chromaticity change (ΔH)</th>
<th>Image information entropy (H)</th>
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From the above experiment results, we can see that the image overall visual effect after the algorithm enhancement in the paper is effectively improved, and the image histogram has more rich details at the same time, thus more meaningful image characteristics can be extracted. Based on objective data analysis in Table 1, 2, with better color maintaining performance and on the premise of ensuring the contrast and brightness enhancement, the image chromaticity processed changes considerably, thus good effect of chromaticity maintaining effect is obtained, the image color after processing is not slant gray, and the detail is outstanding, the change of the brightness and contrast is very obvious, but the operation time is longer and the operating efficiency remains to be further improved.

5. CONCLUSIONS

Image enhancement technology is an important step in the preprocessing part of the digital image processing system. Digital image enhancement results directly affect the image’s advanced processing and interpretation. Based on the analysis and summarization of the existing image enhancement technology, this paper proposes a color image enhancement method based on fuzzy sets. The experimental results show that after the image enhancement by adopting the algorithm in this paper, the image quality is improved to a certain extent, and the image information amount is improved when compared with the original image data, thus better achieving the goal of the color image enhancement.

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