**Decision-making of Selectable Process Plans Based on Petri net with Manufacturing Constraints**

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**Abstract:** Intelligent Computer-aided process planning and decision making for manufacturing systems is a critical subject, that some might argue, has not received the attention it should have from the research community. Despite the progress made in the area of artificial intelligence, there has not been a major step forward in the area of intelligent computer-aided process planning. Petri nets provide a powerful tool to characterize, model and analyse discrete events dynamic systems. However, the efforts on process planning in machinery manufacturing systems have very limited use in practise. This paper presents a new Petri net-based methodology to provide process planning where the manufacturing constraints are explicitly incorporated. Initially a Petri net process planning model is developed. The manufacturing constraints are then incorporated with the Petri net model to generate the corresponding T-invariant, namely the non-negative integer solution of a linear equation set. Two examples verify the effectiveness of the proposed method indicating that the proposed T-invariant decision-making approach can be used to achieve desired optimal/suboptimal process plans for a machinery manufacturing system.

**Key words:** Process planning, Petri nets, T-invariant generation, Manufacturing constraints

1. INTRODUCTION

Intelligent computer-aided process planning of manufacturing systems has challenged researchers since 1965 when Niebel first proposed computer-aided process planning (CAPP). Essentially the aim of CAPP, with regard to the manufacture of products and parts, is to determine an optimal process (in terms of cost and/or time) by configuring and integrating the manufacturing resources. The process planning is constrained by the actual environment and the range and availability of manufacturing resources. The performance of the manufacturing system, in terms of throughput and cost, will ultimately determine the viability and profitability of the manufacturing system. Process planning is not only a knowledge-based decision-making process, but also an optimization process as well where the environmental constraints also have to be considered (Pu, 1997).

Petri nets provide a powerful tool to model and analyse discrete event dynamic systems, like manufacturing systems, as well as systems that also display concurrent, parallel, simultaneous, synchronous, distributed, and resource sharing behaviour. Petri nets were introduced by Petri in 1962 and since the 1980’s have been applied in process related design, for both knowledge representation and decision-making modelling (Tabak, 1985; Garg, 1991). The power of the Petri nets approach lies not only in its ability to provide a rigorous mathematical definition coupled with an intuitive graphical representation, but is also linked to the availability of many accompanying systematic description methods and systematic behaviour analysis technologies.

Petri nets have been applied in manufacturing systems, including the modelling and analysis of discrete manufacturing systems (Liu, 2009; Tüysüz, 2010; Dotolia, 2008), workflow modelling and analysis (Dong, 2005), manufacturing execution system (Mo, 2015) and re-manufacturing processes (Wang, 2015). In more recent studies, Petri nets-based scheduling methods for flexible manufacturing systems have been presented in (Huang, 2012; Baruwa, 2014) and further developed in (Huang, 2010). These studies have demonstrated the good performance and the popularity of Petri nets for manufacturing systems. However, only a few attempts focused on the application of Petri nets in intelligent CAPP systems for process planning and setup planning. Kruth and Detand used Petri nets to develop a CAPP system for nonlinear process planning, which increases the flexibility of scheduling activities. The presented approach is based on the features extracted from CAD software via a neutral file. Until now, there is no satisfied solution for this process. Kiritsis and Porchet proposed a dynamic process planning method based upon a Petri-net model. The same team further introduced a PP-net (Process Planning net) to represent manufacturing knowledge in the form of precedence constraints with machining costs also being incorporated into their PP-net. The way to determine the machining processes and precedence relationship was according to design entities of the machinery part, which limits the further development of the approach. There are also some process planning applications based on fuzzy Petri nets. The theory of fuzzy
Petri nets has defects that some core parameters in the process planning have to be determined based on the subjective experience of individuals or experts. This limits the application of fuzzy Petri nets in CAPP systems. Despite the significant progress made in the area of artificial intelligence in the last 25 years, there is no complete solution by now that integrates all of the variables, which impacts on the performance of the manufacturing system.

This paper presents a new Petri nets-based process planning method, in which machining quality is used to determine the machining processes and precedence relationships. It not only provides Petri nets models for all the functions in the system, but also has the explicit consideration of the actual manufacturing constraints when applying the theory in practise. The Petri net principle is used in conjunction with production regulation ideas, to initially develop a regular Petri net process planning model. The defined process planning model is then considered with both the process operating constraints and equipment constraints, to generate a T-invariant (non-negative integer solution of a linear equation set) for selectable process planning. The most efficient process plan could be then selected based on the actual condition of the workshop or factory. This new approach to T-invariant generation provides the theoretical background for the subsequent decision-making processing sequence. Two examples to verify the effectiveness of the proposed method are presented indicating that the proposed T-invariant decision-making approach can be used to achieve desired optimal/suboptimal process plans for a machining manufacturing system.

2. PETRI NET-BASED SYSTEM AND THE T-INARIANT

2.1. Petri net Finite P/T-System

Definition 1: A Petri net finite P/T-System PN is defined as a six-tuple, which is shown below:

\[ PN=(P; T; F, K, W, M_0) \] (1)

where, P and T are PN's finite place set (P) and finite transition set (T), respectively, and \( P \cap T = \emptyset, P \cup T \neq \emptyset \);

\( F \) represents the flow relation of PN and \( F \subseteq (P \times T) \cup (T \times P) \), dom \( (F) \cup \text{cod}(F) = P \cup T \);

\( K \) is the capacity function of PN, i.e. \( K: P \rightarrow \{1, 2, ..., \} \);

\( W \) is the general weight function \( \iff (x \in PN, i.e. W:(P \times T) \cup (T \times P) \rightarrow \{0,1,2,...\} \) and \( W(x, y)=0, y \notin F \);

\( M_0 \) represents the initial marking of PN. \( M_0: P \rightarrow \{0,1,2,...\} \) and \( \forall p \in P, M_0(p) \leq K(p) \).

Transition \( t \in T \) is enabled at the marking \( M \), expressed as \( M[t > \), with its necessary and sufficient condition being:

\[ \forall p \in P: W(p, t) \leq M(p) - W(t, p) \] (2)

Transition \( t \in T \) transfers from the marking \( M \) to a new marking \( M' \), expressed as \( M[t > M' \), with the new marking \( M' \) satisfying

\[ \forall p \in P: M'(p) = M(p) - W(p, t) + W(t, p) \] (3)

Transition \( t \in T \) is enabled at the marking \( M \), expressed as \( M[t > \), with its necessary and sufficient condition being:

\[ \forall p \in P: W(p, t) \leq M(p) - W(t, p) \] (4)

Transition \( t \in T \) transfers from the marking \( M \) to a new marking \( M' \), expressed as \( M[t > M' \), with the new marking \( M' \) satisfying

\[ \forall p \in P: M'(p) = M(p) - W(p, t) + W(t, p) \] (5)

2.2. Incidence Matrix and State Equation

Definition 2: PN represents a finite Petri net, \( P=\{p_1, p_2, ..., p_n\} \). With \( T=\{t_1, t_2, ..., t_m\} \), the PN's net pattern \( P; T; F \) can be expressed using a so-called incidence matrix \( C_{inc} \), whose elements are defined as:

\[ C_{inc}(p_i, t_j) = W(t_j, p_i) - W(p_i, t_j) \], where \( c_{ij}^+ = W(t_j, p_i) \) represents the weight of arc from the transition \( t_j \) to the output place \( p_i \), and \( c_{ij}^- \leq K(p_i) (i=1,2,...,n); c_{ij}^- = W(p_i, t_j) \) represents the weight of arc from the input place \( p_i \) to the transition \( t_j \) and \( c_{ij}^- \leq M(p_i) (j=1,2,...,m) \).

The incidence matrix, \( C_{inc} \), represents the net pattern in linear algebra form. A row in the matrix represents a place set \( P \) while a column in the matrix represents a transition set \( T \). When a directed arc is used to connect from a transition to a place, the corresponding matrix element is “\( c_{ij}^- \)”. Conversely, when the connection is from a place to a transition, the corresponding matrix element is “\( c_{ij}^+ \)”. If there is no connection, the element is “0”.

According to the transition rule (Huang,2010), \( c_{ij}^+ \), \( c_{ij}^- \) and \( c_{ij} \) are respectively, the increment, decrement, and variation of Token in the place \( p_i \) when transition \( t_j \) is firing.

If the marking of the Petri net is expressed with a non-negative integer vector, according to the incidence matrix, the event \( \tau \in \mathbb{M} \) can be expressed as a matrix equation

\[ M_0 + C \tau \]
where \( M_0 \) and \( M \) are column vectors of the place set \( P \) that is used as the sequence-tag set in \( PN \); \( \tau \) is the transitional sequence in the transforming of \( M_0 \) to \( M \); and \( J \) is the row vector of the transitional set \( T \) that is used as the sequence-tag set in \( PN \). \( J \) is called firing count vector. For \( t_j \in T \), \( J(t_j) \) represents the number of times component \( t_j \) appears in the transitional sequence \( \tau \). The equation is referred to as the state equation of \( PN \).

Murata has demonstrated that all the markings \( M \in R(M_0) \) are necessarily sufficient to meet the state equation, where \( R(M_0) \) is the set of all the reachable markings from \( M_0 \). But the state equation only satisfies the necessary condition, i.e. if \( M \) is reachable from \( M_0 \), \( M \) must meet the state equation; conversely, when \( M \) meets the state equation, it is not necessarily reachable from \( M_0 \). Furthermore, when \( M \) does not meet the state equation, it is definitely not reachable from \( M_0 \).

2.3. \( T \)-invariant

Definition 3: A Petri net \( PN \), \( P=\{p_1, p_2, ..., p_n\} \), \( T=\{t_1, t_2, ..., t_m\} \), \( C_{inv} \) is an incidence matrix of \( PN \). If there is an \( m \)-dimension non-negative integer vector \( J \) with nontrivial solution this then satisfies the following equation

\[
C_{inv} \cdot J_{inv}=0, \quad n=|P|, \quad m=|T|
\]

indicating \( J \) is a \( T \)-invariant of \( PN \).

According to the Definition 2 and Definition 3, it is known that a \( T \)-invariant is a non-negative integer solution of a linear equation set with integer coefficients (except the zero-vector). Therefore, a \( T \)-invariant represents a transitional sequence \( t_1, t_2, ..., t_k \) starting from the initial marking \( M_0 \) in the net system then transmitting to the subsequent markings \( M \), and through the transitional sequence then back to the initial marking \( M_0 \). The existence of the \( T \)-invariant means that the net system is capable of representing the system state, with the values of each component in the \( T \)-invariant determining the occurrence frequency of each transition.

3. THE RULE-BASED PETRI NET PROCESS PLAN MODEL

3.1. Representation and Reasoning of Petri net-Based Rules

The general form of the production rule is IF \( L \) \( \{p_1, p_2, ..., p_n\} \) THEN \( q \), \( q \in \{q_1, q_2, ..., q_m\} \), where \( L=\{\wedge, \lor\} \) represents the logic combination pattern, i.e. combined premises or conclusion can be obtained by combination of premises or conclusions through \( \wedge \) or \( \lor \). In the decision-making system and knowledge representation, there are in general 4 types of production rules as indicated below:

Type 1: IF \( p_1 \wedge p_2 \wedge ... \wedge p_n \) THEN \( q \)
Type 2: IF \( p \) THEN \( q_1 \vee q_2 \vee ... \vee q_m \)
Type 3: IF \( p \) THEN \( q_1 \wedge q_2 \wedge ... \wedge q_m \)
Type 4: IF \( p \) THEN \( q_1 \wedge q_2 \wedge ... \wedge q_m \)

Comparative analysis has shown that there is a specific relation between rule-based methods and the Petri net-based methods. If a production rule system is mapped to a Petri net system, the input place and the output place represent the premise and the conclusion, separately. The token represents a current event, a marking represents an event place, and the marking change means updating the event place. The transition represents the logic combination pattern, i.e. combined premises or conclusion can be obtained by combination of premises or conclusions through \( \wedge \) or \( \lor \). In the decision-making system and knowledge representation, there are in general 4 types of production rules as indicated below:

Definition 4: A rule-based Petri net system for process plans \( PM_{PN} \) is defined as a five-tuple:

\[
PM_{PN}=(P,T,I,O,M)
\]

where, \( P \) is \( PM_{PN} \)'s finite place set, \( P=\{P_n \cup P_c\} \) in which \( Pm \) represents the processing place subset and \( Pc \) is the constraint place subset;

\( T \) is a finite transitional set of \( PM_{PN} \);

\( I \) is the input function of \( PM_{PN} \), \( I:P \times T \rightarrow \{0,1\} \);

\( O \) is the output function of \( PM_{PN} \), \( O:T \times P \rightarrow \{0,1\} \);

\( M \) is the finite marking set of \( PM_{PN} \), \( M=\{M_m \cup M_c\} \rightarrow \{0,1\} \), where \( M_m \) is the marking corresponding to \( P_m \) and \( M_c \) corresponds to \( P_c \).

Based on Definition 4, Figure 1. shows a Petri net model both before and then after a transition has been activated. The input place \( p_n \) changes from having a token to having no token and the output place \( p_n \) from no token to having a token, while the input place \( p_n \) always has a token providing "self-circulation" regardless of the transition occurrence. This indicates that when the process constraints and the equipment constraints are met, the processing in the model is repeatable. Because the Petri net is equipped with this feature, a number of process plans can be obtained for one manufacturing feature.
3.2. The T-invariant Method for Generation of Process Plans

According to the principle of the Petri net, it is known that algebraic calculation of T-invariant can be used in the simple net ($\forall x \in P \times T, x \neq 0$) only. Because the model shown in Figure 1, has a "self-loop", the model should be modified and transformed. The Petri net is simplified by changing the bi-directional arrows with self-loop into single-directional arrows pointing at the transition, while removing all constraint places and disabled transitions after the Petri net system meets the operational conditions. The procedure for the generation of the processing alternatives consists of the following 13 steps:

Step 1 Establish a Petri-net model for processing alternatives based on the rule set and the mapping relationship between the rule and Petri net;

Step 2 Let all constraint places, that meet the machinery parts' constraint conditions, generate a token to obtain the initial marking $M_0$ of the constraint place.

Step 3 Let the processing place $p_{n0}$ generate a token with all other places being zero to obtain the initial marking $M_{n0}$ of the processing place, as below:

$$M_{n0}(p_{n0}) = 1$$

$$M_{n0}(p_{m}) = 0, \quad j \geq 1$$

Step 4 Based on the system initial marking $M_0 = \{M_{n0} \cup M_0\}$, the enabled transition set $T_e$ is obtained with it being assumed $T_{ne} = T_e$;

Step 5 If $T_o = \emptyset$, the algorithm is ended; if $T_o \neq \emptyset$, then go to Step 6;

Step 6 Trigger all transitions in $T_{ne}$ and calculate the subsequent marking $M$, to obtain:

$$M(p_i) = M(p_i) + O(p_i) - I(p_i), \quad \forall p_i \in P, T_e$$

Step 7 Based on the subsequent marking $M$, the enabled new transition set $T_{ne}$ is obtained. If $T_{ne} = \emptyset$, go to Step 8; if $T_{ne} \neq \emptyset$, let $T_e = \{T_e + T_{ne}\}$ and go back to Step 6;

Step 8 Simplify the Petri net model using $M_0$ and $T_e$:

1. Change all the bi-directional arrows in the model as one-way arrows being directed at the transition;
2. Delete all the constraint places in the model;
3. Delete transitions that did not appear in $T_e$ and associated output places in the model;

Step 9 Add a virtual transition $T_v$, between the initial place and all end places to form a simplified Petri net model that can operate cyclically for a required number of times;

Step 10 $M_0$ and $T_e$ are used to calculate the incidence matrix $C$;

Step 11 The equation $C \cdot J = 0$ is used to calculate all T-invariants (except the all-zero vectors);

Step 12 Each T-invariant is used to solve the corresponding place sequences; each place sequence is a process plan and all the process plans constitutes a process plan set PM;

Step 13 In consideration of the real-time constraints within the workshop, an optimized process plan from the set PM is selected as the actual process plan.

4. APPLICATIONS

In the field of mechanical engineering, there is no clear standard that could be applied for proofing the correctness of the process planning method. Thus the presented method will be valid in real applications, and the results will be compared to the process plan made by the experience manufacturing process designers, which will be described in this section.

Process planning to provide processing of the outer surface of a cylindrical part and also processing of a flat surface are considered. The processing is needed to satisfy the requirement for an adequate smooth finish to the part. These particular processing examples are used to demonstrate the uses of the T-invariant method to generate process plans via the Petri net approach. Petri-net based process planning for other surfaces of other parts such as inner bores and tooth profiles can also easily be derived.
4.1. Outer Surface of Cylindrical Part

4.1.1. Third-order Headings

Table 1. indicates all the process plans with surface roughness for the outer surface of cylindrical part. These were developed based upon manufacturing technology theory and basic rules relating to the processing of cylindrical part surfaces. It can be seen that different types of grinding and lathing operations are considered. The process constraints relate to the equipment condition and process condition defined for each type of processing operation. The parameters considered in the generation of the process plans are the dimensional tolerance, surface roughness and geometric tolerance.

The Petri net-based process plan model for the processing of the cylindrical part outer surface is obtained according to Definition 4 and Table 1. The model is shown in Figure 2.

<table>
<thead>
<tr>
<th>Rule (t)</th>
<th>Processing conditions</th>
<th>Constraints</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(Blank) pm0</td>
<td>Ra&lt;50, IT≤13, HRC&lt;35</td>
<td>Lathe pc1 Rough-lathing pm1</td>
</tr>
<tr>
<td>2</td>
<td>Rough-lathing pm1</td>
<td>Ra≤6.3, IT≤10, HRC&lt;35</td>
<td>Lathe pc2 Semi-fine-lathing pm2</td>
</tr>
<tr>
<td>3</td>
<td>Semi-fine-lathing pm2</td>
<td>Ra≤1.6, IT≤8, HRC&lt;35</td>
<td>Lathe pc3 Fine-lathing pm3</td>
</tr>
<tr>
<td>4</td>
<td>Fine-lathing pm3</td>
<td>0.05≤Ra≤0.8, 6≤IT≤7, HRC&lt;35</td>
<td>Roller press pc4 Rolling pm4</td>
</tr>
<tr>
<td>5</td>
<td>Semi-fine-lathing pm2</td>
<td>0.4≤Ra≤0.8, 6≤IT≤7, Part≠non ferrous</td>
<td>Grinder pc5 Grinding pm5</td>
</tr>
<tr>
<td>6</td>
<td>Semi-fine-lathing pm2</td>
<td>Ra≤1.6, IT≤9, Part≠non ferrous</td>
<td>Grinder pc6 Rough grinding pm6</td>
</tr>
<tr>
<td>7</td>
<td>Fine-lathing pm3</td>
<td>Ra≤0.4, IT≤6, Part≠non ferrous</td>
<td>Grinder pc7 Fine grinding pm7</td>
</tr>
<tr>
<td>8</td>
<td>Rough grinding pm8</td>
<td>Ra≤0.4, IT≤6, Part≠non ferrous</td>
<td>Grinder pc8 Fine grinding pm8</td>
</tr>
<tr>
<td>9</td>
<td>Fine grinding pm9</td>
<td>0.012≤Ra≤0.1, IT≤5, Part≠non ferrous</td>
<td>Super-finishing machine pc9 Super-finishing pm9</td>
</tr>
<tr>
<td>10</td>
<td>Fine-lathing pm10</td>
<td>0.012≤Ra≤0.1, 5≤IT≤6, Part≠non ferrous</td>
<td>Diamond lathe pc10 Diamond-lathing pm10</td>
</tr>
<tr>
<td>11</td>
<td>Fine grinding pm11</td>
<td>0.006≤Ra≤0.025, IT≤5</td>
<td>Mirror-face grinder pc11 Mirror-face grinding pm11</td>
</tr>
<tr>
<td>12</td>
<td>Fine grinding pm12</td>
<td>0.05≤Ra≤0.1, IT≤5</td>
<td>Lapping machine pc11 Lapping pm11</td>
</tr>
</tbody>
</table>

Note: Ra, IT, and HRC represent Surface Roughness, International Tolerance, and Rockwell Hardness measured on the C scale.
4.1.2. Real Case Analysis

Figure 3. (a) is a cross-section drawing of a cylindrical part on which the outer surface processing with the roughness of 0.4 should be carried out. According to the Petri net-based alternative process plan model in Figure 2, process plans that satisfy the constraints of this machinery part can be derived. This is depicted in Figure 3. (b). The addition of the virtual transitions \( t_{v1} \) and \( t_{v2} \), provides a simplified Petri net-based alternative process plan model for cylindrical surface processing, which is shown in Figure 3. (c).

From Definition 2, the incidence matrix \( C_{7 \times 9} \) is obtained.

\[
\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\
\end{bmatrix}
\]

The equation \( C \cdot J = 0 \) in Definition 3 is used to calculate all the \( T \)-invariants. The solutions of the 3 \( T \)-invariants obtained in this particular example (the calculation process is omitted) are:

\[
J_1 = (1, 1, 0, 1, 0, 0, 0, 1, 0)^T
\]

\[
J_2 = (1, 1, 1, 0, 0, 0, 1, 0, 1)^T
\]

\[
J_3 = (1, 1, 0, 0, 1, 0, 1, 0, 1)^T
\]

Applying \( J_1 \), \( J_2 \), and \( J_3 \) to Table 1, three alternative process plans are derived for processing the cylindrical outer surface of the machinery part.

1. Rough lathing \((p_{m1})\) – Semi-fine lathing \((p_{m2})\) – grinding \((p_{m5})\)
2. Rough lathing \((p_{m1})\) – Semi-fine lathing \((p_{m2})\) – fining lathing \((p_{m3})\) – fine grinding \((p_{m7})\).
3. Rough lathing \((p_{m1})\) – Semi-fine lathing \((p_{m2})\) – rough grinding \((p_{m6})\) – fine grinding \((p_{m7})\).

Figure 3. (a) The cylindrical part to be processed; (b) The alternative process plan model that satisfies the constraints in (a); (c) The simplified model of (b).

Based on the actual production conditions in the workshop, e.g. availability of the equipment and/or the time/cost taken to process the surface, one of the three alternative process plans could be selected to process the
outer cylindrical surface of the machinery part, while the remaining 2 plans could be used as alternatives.

4. 2. Flat Surface

Similar to the first example, Table 2. indicates all the process plans for the processing of a flat surface. Using this table in conjunction with Definition 4, the Petri net-based process plan model is generated. This is shown in Figure 4. Applying the model to a real machinery part is depicted in Figure 5.(a), while the process plans that satisfy the constraints are shown in Figure 5.(b). After the addition of the virtual transitions tv1, tv2, tv3, and tv4, the simplified Petri net-based alternative process plan model for flat surface processing is obtained. This is shown in Figure 5.(c).

### Table 2. The Flat Surface Process Plans

<table>
<thead>
<tr>
<th>Rule (d)</th>
<th>Processing conditions</th>
<th>Equipment conditions</th>
<th>Subsequent processing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise</td>
<td>Constraints</td>
<td>Symbol</td>
<td>Symbol</td>
</tr>
<tr>
<td>1 (Blank)</td>
<td>$p_{n0}$ $Ra\leq50$, $IT\leq13$, $HRC&lt;35$, Part = end surface</td>
<td>Lathe $p_{r1}$</td>
<td>Rough-lathing $p_{m0}$</td>
</tr>
<tr>
<td>2 Rough-lathing</td>
<td>$p_{m1}$ $Ra\leq6.3$, $IT\leq9$, $HRC&lt;35$, Part = end surface</td>
<td>Lathe $p_{r2}$</td>
<td>Semi-fine-lathing $p_{m2}$</td>
</tr>
<tr>
<td>3 Semi-fine-lathing</td>
<td>$p_{m2}$ $Ra\leq1.6$, $IT\leq8$, $HRC&lt;35$, Part = end surface</td>
<td>Lathe $p_{r3}$</td>
<td>Fine-lathing $p_{m3}$</td>
</tr>
<tr>
<td>4 Semi-fine-lathing</td>
<td>$p_{m2}$ $Ra&lt;0.8$, $6\leq IT \leq7$, Part = end surface</td>
<td>Grinder $p_{r4}$</td>
<td>Grinding $p_{m4}$</td>
</tr>
<tr>
<td>5 (Blank)</td>
<td>$p_{n0}$ $Ra\leq50$, $IT\leq13$, $HRC&lt;35$, Part $\neq$ end surface</td>
<td>Milling machine or planer $p_{r5}$</td>
<td>Rough-milling or rough-planing $p_{m5}$</td>
</tr>
<tr>
<td>6 Rough-milling or rough-planing</td>
<td>$p_{m5}$ $Ra\leq6.3$, $IT\leq9$, $HRC&lt;35$, Part $\neq$ end surface</td>
<td>Milling machine or planer $p_{r6}$</td>
<td>Fine-milling or fine-planing $p_{m6}$</td>
</tr>
<tr>
<td>7 Fine-milling or fine-planing</td>
<td>$p_{m6}$ $Ra\leq0.8$, $IT \leq7$, Part $\neq$ end surface</td>
<td>Grinder $p_{r7}$</td>
<td>Grinding $p_{m7}$</td>
</tr>
<tr>
<td>8 Fine-milling or fine-planing</td>
<td>$p_{m6}$ $Ra\leq0.8$, $IT \leq7$, Part $\neq$ end surface</td>
<td>Grinder $p_{r8}$</td>
<td>Rough grinding $p_{m8}$</td>
</tr>
<tr>
<td>9 Rough-grinding</td>
<td>$p_{m8}$ $Ra\leq0.4$, $IT \leq7$, Part $\neq$ end surface</td>
<td>Grinder $p_{r9}$</td>
<td>Fine-grinder $p_{m9}$</td>
</tr>
<tr>
<td>10 Fine-milling or fine-planing</td>
<td>$p_{m6}$ $0.1\leq Ra \leq0.8$, $IT \leq7$, $HRC&lt;35$, Part $\neq$ end surface</td>
<td>Vise bench $p_{r10}$</td>
<td>Scraping $p_{m10}$</td>
</tr>
<tr>
<td>11 Fine-milling or fine-planing</td>
<td>$p_{m6}$ $0.2\leq Ra \leq0.8$, $IT \leq7$, $HRC&lt;35$, Part $\neq$ end surface</td>
<td>Planer $p_{r11}$</td>
<td>Wide edge fine-planing $p_{m11}$</td>
</tr>
<tr>
<td>12 Grinding</td>
<td>$p_{m7}$ $0.025 \leq Ra \leq0.2$, $IT \leq6$, Part $\neq$ end surface</td>
<td>Lapping machine $p_{r12}$</td>
<td>Lapping $p_{m12}$</td>
</tr>
<tr>
<td>13 Lapping</td>
<td>$p_{m12}$ $0.012 \leq Ra \leq0.1$, $IT \leq5$, Part $\neq$ end surface</td>
<td>Polishing machine $p_{r13}$</td>
<td>Polishing $p_{m13}$</td>
</tr>
</tbody>
</table>

**Note:** $Ra$, $IT$, and $HRC$ represent Surface Roughness, International Tolerance, and Rockwell Hardness measured on the C scale.

![Figure 4. The Petri net-Based Alternative Process Plan Model for Flat Surface Processing](image-url)
According to Definition 2 and Definition 3, the following matrix \( C_{8 \times 11} \) and 4 T-invariants solutions are obtained.

\[
J = \begin{pmatrix}
t_5 \\
t_6 \\
t_7 \\
t_8 \\
t_{10} \\
t_{11} \\
t_{12} \\
t_{13} \\
t_{14}
\end{pmatrix}^T
\]

\[
P_{m0} = \begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1
\end{pmatrix}
\]

\[
P_{m5} = \begin{pmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
P_{m6} = \begin{pmatrix}
0 & 1 & -1 & -1 & 0 & -1 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
P_{m7} = \begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0
\end{pmatrix}
\]

\[
P_{m8} = \begin{pmatrix}
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
P_{m9} = \begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0
\end{pmatrix}
\]

\[
P_{m10} = \begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0
\end{pmatrix}
\]

\[
P_{m11} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1
\end{pmatrix}
\]

Correspondingly, by applying \( J_1, J_2, J_3 \) and \( J_4 \) to Table 2, four alternative process plans for processing the flat surface are derived. As in the previous example, a consideration of the actual production conditions in the workshop will help in the selection of the plan to be implemented.

1. Rough milling or rough planing \((p_{m5})\) – fine milling or fine planing \((p_{m6})\) – grinding \((p_{m7})\)
2. Rough milling or rough planing \((p_{m5})\) – fine milling or fine planing \((p_{m6})\) – rough grinding \((p_{m7})\) – fine grinding \((p_{m7})\)
3. Rough milling or rough planing \((p_{m5})\) – fine milling or fine planing \((p_{m6})\) – scraping \((p_{m10})\)
4. Rough milling or rough planing \((p_{m5})\) – fine milling or fine planing \((p_{m6})\) – wide edge fine-planing \((p_{m11})\)

Figure 5. (a) The Machinery Part to be Processed; (b) The Alternative Process Plan Model That Satisfies The Constraints in (a); (c) The Simplified Model of (b).

5. DISCUSSION AND CONCLUSION

The machine manufacturing system is a typical discrete event dynamic system, in which modelling and analysis methods such as queue networks, activity cycle diagrams, and Max-algebra are commonly applied. Unfortunately these methods are not suitable for decision-making with regard to selectable process plans. There have been several studies attempt to indicate the capability of Petri net-based methods to achieve such task, but no sufficient solution have been provided yet. In Kiritsis and Porchet’s method, the machining processes and
manufacturing procedures are determined by the design entities, which is according to the constructive solid geometry that forms the shape of the machinery part. The method does not refer to how to determine the processing steps of each design entity and the precedence relationships of the design entities. The machining quality, such as machining accuracy and surface roughness, as well as the operation of thermal treatment are not considered either. However these issues cannot be ignored in process planning. Kruth and Detand presented a Petri nets-based CAPP system for nonlinear process planning. The system requires features extracted from CAD systems and converted via a neutral file. In this process, it must overcome the key issues such as feature extraction, feature combination, feature decompensation, and feature compile. Unfortunately, no efficient solution has been found yet. Furthermore, the process plan is generated over a transformation process from operation-step Petri nets to machine Petri nets, and the data for creating the operation-step Petri nets is from CAD systems. Considering the significant difference between the Petri nets system and CAD system, data transfer between the systems is also an issue. Thus, it is not possible to apply this method directly in practise for process planning. This paper presents a new Petri net-based method for process planning that has a comprehensive consideration of the machining qualities including machining accuracy, surface roughness, and thermal treatment. Comparing with the previous Fuzzy Petri nets-based studies , which require experienced experts to determine core parameters in process planning, this method has significantly overcome this issue. In addition, there is also a common drawback of the above Petri nets applications. When the complexity of the machinery part increases, the number Petri nets elements (place, transition and direct arc) increases significantly. This will result in the net explosion, and thus no useful process plans will be provided. We employ T-invariant in the method that removes the useless connections efficiently. For complex machinery part with increasing number of elements, it is still possible to provide selectable process plans including both optimal and suboptimal ones. The presented method in this paper has more value in practise.

Using the developed processing planning approach that incorporates constraints explicitly into the T-invariant generation, alternative process plans were produced for the processing of a cylindrical part surface and also a flat surface. The parameters considered in the generation of the process plans are the dimensional tolerance, surface roughness and geometric tolerance. Since there is no standard that can be used to valid the method, we compare the Petri-net based process plans developed in the examples with these are implemented by experienced manufacturing process designers. It has comparable results. This shows that the developed Petri-net approach is a practical as well as an efficient tool for producing a manufacturing process plan.

Although the machinery parts in the example are fairly simple parts, with only a few processes to provide surface finishing, it does not invalidate the applicability of the method. The emphasis here was to illustrate the processing planning design approach in as transparent a way as possible while still retaining some resemblance to practical manufacturing part surface finishing. The purpose of the examples is to proof that the presenting method is able to generate multiple process approaches even for such simple parts. Then according to the actual manufacturing constrains in the workshop or factory, the most suitable approach can be further selected. Complex parts, with their range of possible processing operations and constraints can also be addressed via the proposed approach. But it will generate complex process plans, which will be difficult to demonstrate in this paper. For proofing the correctness of the method at this stage, a simple part or a complex part will show the same result. For simple machinery parts, the process plan can be conducted directly from the model (e.g. Figure 3. and Figure 5.). The more complicated the part structure it is, the more manufacturing processes it will require. The subsequent process planning model will increase in size, containing more states and transitions. For very large models it will be become more difficult to implement the process plan directly from the model. Future work will investigate these cases with a view to combining the Petri net modelling method with the eM-Plant simulation method.

Process planning has been treated as the key factor of the integration from the design process to manufacturing . Many of the commercial available software or models are still not able to meet the practical requirements from real applications. During the last two decades, much effort has been expended to understand and incorporate the mainly heuristic approach taken by process designers into automatic manufacturing system decision-making and process planning tools. The presenting method is a new attempt of the decision-making method for the intelligent CAPP. As mentioned by Xu. “Optimality is the Holy Grail for setup planning researchers”, the same applies to the decision-making of selectable process plans.

REFERENCES


