Management Efficiency Evaluation Model Based on High-dimensional Index DEA Algorithm

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Abstract
Management efficiency evaluation is conducive to enhance the core competitiveness of enterprises, so as to better meet the individual needs of customers and effectively reduce the loss of customers. The management efficiency evaluation model based on the high-dimensional index DEA algorithm was proposed in this paper. First the efficiency evaluation model which is applicable to the economics was constructed according to the classic DEA algorithm and PCA model was introduced to realize the transformation of the input space to the feature space by the nonlinear mapping. Then the mapping data was conduct the linear PCA and the input and output high-dimensional indicators of the DEA model was differentiated optimized by using the PCA and the correlation between indicators were eliminated to obtain the low dimensional input and output principal component indicators. The example simulation results showed that the improved model in this paper has a good evaluation effect and obtains the correct evaluation results of customer relationship management efficiency for the testing company, which is worthy to be popularized.

Key words: Customer Relationship Management, Efficiency Evaluation, PCA-DEA, Kernel Function Optimization, Differential Optimization.

1. INTRODUCTION

In recent years, the research on the theory of the customer relationship management has attracted more and more attention. The information age has witnessed the rapid development of the Internet and e-commerce and the profound changes of all kinds of enterprises all over the world (Liu, 2015). The enterprises are facing the great changes of the business environment and the traditional business model is increasingly challenged. In particular, the current homogenization of products and services in many industries is becoming increasingly significant and it is also getting more and more difficult for the enterprises to establish competitive advantage (Xiong and Wei, 2015). Many enterprises, especially those in the United States and other developed countries, treat the customer resource as the most important resource and the customer relationship management as a magic weapon. Through the implementation of customer relationship management, they further take series of measures to obtain the sustainable competitive advantage and realize the enterprise's sustainable development like improving the core competitiveness of enterprises, meeting customer demand, improving customer satisfaction, loyalty and customer value and maintaining a long-term good relations of cooperation with the customers (Dai and Zhang, 2015).

Throughout the domestic and international academic circles, many scholars have studied the customer relationship management from different aspects. Russell Viner presented the customer relationship management strategy implementation model, including seven aspects which are creating customer database, customer analysis, choosing customer, establishing customer orientation, relationship marketing, protecting customer privacy and indicators evaluation, respectively, and he also discussed the realization method and the technical plan of the implementation aspects (Russell, 2011). Chalmeta put forward the IRIS methodology of management and technical integration of customer relationship management containing six parts which are the framework of the customer relationship management, customer strategy, process planning, customer relationship evaluation system, implementation and monitoring, respectively, and he also proposed the improvement scheme on the customer relationship management process based on the customer relationship management best practice (Chalmeta, 2012). In the customer segmentation field, Su-Yeon Kim et.al established the customer segmentation model based on the customer life value theory and they showed the specific case of customer differentiation strategy for customer segmentation (Kim, 2014). In the field of the customer relationship management performance evaluation, Jonghyeok Kim established the evaluation model of customer relationship management and he applied the system structure and thinking method of the Balanced Scorecard (BSC) to the customer relationship management evaluation including selecting evaluation index, applying process cause and effect analysis and establishing evaluation process system and so on, in which the evaluation index contains four aspects, namely, customer knowledge index, customer interaction index, customer value index and customer satisfaction index (Kim, 2013). In the research of system structure, Michael Fayerman put...
forward the IRIS computer system model of the customer relationship management in which the customer relationship management system is divided into the operation level, the tactical level and the strategy level, including five major functions which are marketing management function, sales management function, after sales management function, human resource management function and financial management function, respectively (Michael, 2012). In the study of system function, Meta Group divided the customer relationship management system into three levels, which are the operation level, covering the operational functions of the front desk; the analysis level covering the analysis function of the background and the collaboration level, covering customer interaction function, respectively, establishing the framework for customer relationship management system function. Marianna Sigala thought the customer relationship management system could be more effective in integrating information and communication technology in the hotel industry and pointed out that the successful implementation of customer relationship management system required the combination of three aspects of the ICT, the internal and external relations and the knowledge management (Marianna, 2005). Gurau proposed that the online program could accurately analyze the customer profile for the customer segmentation and locate the target customers, so the customer relationship management is one of the important enterprise strategies of the e-commerce company (Gurau, 2013). He studied the implementation process of the customer relationship management system in the online retail industry and summarized the implementation system of the customer relationship management system from the two perspectives of the process optimization and the organizational change.

A management efficiency evaluation model was proposed based on high-dimensional index differential optimized DEA algorithm in this paper, aiming at the needs of the company’s customer management efficiency evaluation. And simulations were carried out to verify the effectiveness of the proposed method.

2. THE EFFICIENCY EVALUATION MODEL BASED ON THE DEA ALGORITHM

The data envelopment analysis (DEA) is using the method of linear programming in operational research on the relative effectiveness of decision-making unit between coefficients to make evaluation. In the DEA method, the evaluated unit or department is called the decision-making unit which is denoted by $DMU_i$. The $DMU$ could be a university or an enterprise, or an entity with the same objective and external environment as well as the same type of input and output indicators. If the input vector and the output vector of one $DMU$ are assumed to be $(x_1, ..., x_m)^T$ and $(y_1, ..., y_n)^T$, respectively, the set $T = \{(x, y)\}$ is the production possibility set composed of all production activities. If the number of input, output and each decision-making unit are $m$, $s$ and $n$, respectively, the input vector and the output vector of $DMU_i$ are as follows

$$
\begin{align*}
  x_j &= (x_{1j}, x_{2j}, ..., x_{mj})^T > 0, j = 1, 2, ..., n \\
  y_j &= (y_{1j}, y_{2j}, ..., y_{nj})^T > 0, j = 1, 2, ..., n
\end{align*}
$$

(1)

and the input weight and the output weight are as follows

$$
\begin{align*}
  v &= (v_1, v_2, ..., v_m)^T \\
  u &= (u_1, u_2, ..., u_s)^T
\end{align*}
$$

(2)

each decision-making unit has its corresponding efficiency evaluation index as

$$
\begin{align*}
  h_j &= \frac{u_j^T y_j}{v_j^T x_j} = \frac{\sum_{i=1}^{m} u_i y_{ij}}{\sum_{i=1}^{n} v_i x_{ij}}
\end{align*}
$$

(3)

The evaluation model will be constructed with the constraint of the efficiency index of all the decision-making units if the efficiency index of the $j_0$ th is treated as the target which is

$$
\begin{align*}
  \max h_{j_0} &= \frac{\sum_{i=1}^{m} u_i y_{i,j_0}}{\sum_{i=1}^{n} v_i x_{i,j_0}} \\
  \sum_{i=1}^{m} u_i y_{ij} &\leq 1 \\
  \sum_{i=1}^{n} v_i x_{ij} &\leq 1
\end{align*}
$$

(4)
The economic benefit can be regarded as the ratio of the output value and the input value of each enterprise. First, take the single input and the single output as examples, assuming that there are vendors A and B, with the amount of labor L on behalf of the input indicator, output Q on behalf of the output indicator, as shown in figure 1. The economic efficiency of them was compared and it was not hard to find that the slope of A was greater than that of B, which indicated that the efficiency of the manufacturer A was relatively high with

$$\frac{A_0}{A_L} > \frac{B_0}{B_L}$$  \hspace{1cm} (5)

Then the concept of this kind of efficiency was extended to the multi-input and multi-output model system, supposing that there were even factories in the production activities and each factory had two inputs and one output, as shown in Table 1. Now input 1 and input 2 were considered as the transversal vector and the longitudinal vector, respectively, and then the coordinates of the seven manufacturers were marked in the coordinate system and they were further projected on the horizontal plane, as shown in Figure 2.

*Figure 1. Schematic model of single input single output*

<table>
<thead>
<tr>
<th>Table 1. Multiple input multiple output examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision Making Units</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
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<td>4</td>
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<td>6</td>
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<tr>
<td>7</td>
</tr>
</tbody>
</table>

*Figure 2. Input-output schematic*
And the most closet one to the origin O, DMU, was connected and the horizontal line extended by DMU were added to obtain the minimum convex hull composed of a part of the decision-making unit. The minimum convex hull was the production frontier and the locations of the other decision-making units 2 and 4 were both at the upper right of the production frontier. If all the points were connected with the origin, the connection point A of DMU and the origin would be found and it was on the connection of the decision-making unit 1 and 5. After calculation, if the manufacturer wanted to make production in A point to achieve the output of 130, only 13.9 of the input 1 and input 2 would be needed. However, the input 1 and input 2 would be 15 greater than 13.9 when the manufacturer lied in DMU, which indicated that the production process was not relatively effective and it could be improved by using much less input to produce the equal output with DMU. If the relative efficiency value was expressed with the ratio of OA/O2, then the relative efficiency value of DMU would be 0.929 < 1. In the above example, there were only 7 observed values. If the actual observed number of the sample was sufficient, a more smooth curve would be obtained and then the original 5 points in the minimum convex hull might lose their effectiveness, that is to say, probably not all of them lied in the new curve. This indicated that the DEA method was a relative judgment of the efficiency of the observation samples and different observation values would lead to different results.

3. THE MANAGEMENT EFFICIENCY EVALUATION MODEL OF DIFFERENTIAL OPTIMIZED DEA ALGORITHM

3.1 The PCA Algorithm Based on The Kernel Function Optimization

The principal component analysis (PCA) is to select the main indicators from the original index which can generally reflect the information of all the original indicators. Assuming that the number of the samples and the variables of each sample are n and p, respectively, a sample data matrix with \( n \times p \) order will be constructed as

\[
X = \begin{bmatrix}
  x_{11} & x_{12} & \cdots & x_{1p} \\
  x_{21} & x_{22} & \cdots & x_{2p} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{n1} & x_{n2} & \cdots & x_{np}
\end{bmatrix}
\]  

(6)

If the i-th column vector of X is \( X_i (i = 1, 2, \ldots, p) \), then the linear combination of \( X_1, X_2, \ldots, X_p \) of the data matrix X will be expressed as

\[
F_i = a_i X_1 + a_2 X_2 + \ldots + a_p X_p, i = 1, 2, \ldots, p
\]  

(7)

If \( a_i = (a_{i1}, a_{i2}, \ldots, a_{ip})^T \) is established, then \( a_i \) need meet the following formula.

\[
a_i^T = 1, i = 1, 2, \ldots, p
\]  

(8)

and \( a_i \) is the unit vector which is determined by the following two principles. One is that \( F_i, F_j (i \neq j, i = 1, 2, \ldots, p) \) has no correlation namely

\[
\text{cov}(F_i, F_j) = 0
\]  

(9)

And the other one is that if \( F_1 \) and \( F_2 \) represent the maximum variance values of all linear combination of \( X_1, X_2, \ldots, X_p \), respectively, then \( F_1 \) and \( F_2 \) will be called the first principal component, respectively, in which \( F_1 \) has no correlation with \( F_1 \) and p principal components are defined. The amount of information contained in p principal components will be decreased in order and the amount of information contained in each p can be represented by the variance. At the same time the contribution value of the variance of each p equals to the eigenvalue \( \lambda_i \) of the original index and the feature vector of \( \lambda_i \) can be expressed by the combination
coefficient $a_i$. Then the formula for calculating the variance contribution rate of $F_i$ is $\lambda_i / \sum_{i=1}^{p} \lambda_i$ and the greater the value of $\lambda_i / \sum_{i=1}^{p} \lambda_i$, the more the amount of information contained in the principal component.

In this paper, the radial basis function (RBF) was used as the kernel function to establish the reasoning test model. The RBF is

$$K(x, x_i) = \exp \left( -\frac{|x - x_i|^2}{\sigma^2} \right)$$ (10)

In the classification analysis, the final decision function based on the RBF kernel was obtained as

$$M(x) = \text{sgn} \left( \sum_{i=1}^{M} a_i y_i k(x, x_i) + b \right) = \text{sgn} \left( \sum_{i=1}^{M} a_i y_i \exp \left( -\frac{|x - x_i|^2}{\sigma^2} \right) + b \right)$$ (11)

in which $x_i$ is the sample vector of the support vector; $x$ is the predicted factor vector; $a_i$ and $b$ are the coefficients to be determined in the establishment of the PCA model and $\sigma$ is the kernel parameter and the sum operations are only performed on the support vectors. The conversion of input space to the feature space is realized through the nonlinear mapping $\phi$ for the PCA algorithm based on kernel function weighted iterative optimization and then the data of the mapping is carried out with linear PCA, so it has strong nonlinear processing ability. If the M samples in the input space $x_i$ meets $\sum_{i=1}^{M} x_i = 0$ with $x_i \in \mathbb{R}^N$, then its covariance matrix will be as

$$C = \frac{1}{M} \sum_{i=1}^{M} x_i x_i^T$$ (12)

For the general PCA method, the characteristic value of the contribution rate and the corresponding feature vector are obtained by solving the characteristic equation,

$$\lambda v = Cv$$ (13)

If the nonlinear mapping function is introduced to transform the sample point $x_1, x_2, ..., x_M$ in the input space to the sample point $\phi(x_1), \phi(x_2), ..., \phi(x_M)$ in the feature space and the formula

$$\sum_{i=1}^{M} \phi(x_i) = 0$$ (14)

is assumed to be founded, the covariance matrix in the feature space $F$ will be

$$\bar{C} = \frac{1}{M} \sum_{i=1}^{M} \phi(x_i) \phi(x_i)^T$$ (15)

Therefore, the PCA in the feature space is the characteristic value and characteristic vector of the solving equation,

$$\lambda v = \bar{C} v$$ (16)

and then the formula

$$\lambda(\phi(x_i) v) = \phi(x_i) \bar{C} v$$ (17)
is founded. It is noted that \( v \) in the above formula can be represented by the linear \( \phi(x_i) \), namely

\[
v = \sum_{j=1}^{M} a_i(x_i)
\]

If \( M \times M \) matrix \( K \) is defined as

\[
K_{ij} = \phi(x_i)\phi(x_j)
\]

which can be simplified as

\[
M \lambda K\alpha = K^2\alpha
\]

and

\[
M \lambda\alpha = K\alpha
\]

is obvious founded, then the demanded characteristic value and characteristic vector can be obtained by solving the formula (21). The projection of the testing samples in the \( F \) space vector are

\[
(V^t\phi(x)) = \sum_{i=1}^{M} \alpha_i^t\phi(x_i)\phi(x)
\]

in which \( \tilde{K} \) is used to replace \( K \) at this time.

### 3.2 Differential Optimization of The DEA Model Based on The Improved PCA

The Low dimensional principal component indicators of the input and the output were obtained through the differential optimization of the high dimensional indicators of the input and the output in the DEA model using the PCA and they met the conditions of the formula,

\[
\sum_{i=1}^{q} \lambda_i \geq 0.85
\]

Considering that there might be negative in the standardized data obtained from the PCA, the data obtained from the PCA would be normalized to a dimensionless interval \([0.1,1]\) in order to the nonnegative requirements of the DEA analysis on the data of the input and the output indicators and the formula was

\[
Z'_y = 0.1 + \frac{Z_y - b_j}{a_j - b_j} \times 0.9
\]

in which \( a_j \) is the maximum value of the \( j \)-th standardized indicator and \( b_j \) is the minimum value of the \( j \)-th standardized indicator. The weight judgment matrix \( A = (r_{ij})_{q \times q} \), in which \( r_{ij} = \frac{\lambda_i}{\lambda_j} \), of every principal component of the \( DMU \) was obtained by comparing the weight coefficient of every two principal components based on determining the number of principal components and the formula was as the following

\[
A = \begin{bmatrix}
1 & \frac{\lambda_2}{\lambda_1} & \ldots & \frac{\lambda_q}{\lambda_1} \\
\frac{\lambda_2}{\lambda_1} & 1 & \ldots & \frac{\lambda_q}{\lambda_2} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\lambda_q}{\lambda_1} & \frac{\lambda_q}{\lambda_2} & \ldots & 1
\end{bmatrix}
\]
\( \frac{\lambda_i}{\lambda_j} = 1 \) indicated that the j-th principal component indicator was as important as the i-th principal component indicator and \( \frac{\lambda_i}{\lambda_j} > 1 \) showed that the i-th principal component indicator was more important than the j-th principal component indicator. According to the result of the verification, the weight judgment matrix A met the requirements of the consistence because \( r_{ij} = \frac{1}{r_{ji}} \). If \( C_s \) and \( B_s \) respectively represented the judgment matrix obtained through the comparison between every two main components of the input and the output, while \( \lambda_m \) and \( \lambda_s \) represented the maximum value of \( C_s \) and \( B_s \), respectively, the weighted constraint closed convex cone would be obtained based on \( U = \{ u \mid (C_s - \lambda_m F_u) \} \) and \( V = \{ v \mid (B_s - \lambda_s F_v) \} \). Then the evaluation results with the weight constraints of the input and the output indicators could be obtained when the DEA model was used to evaluate the DMU.

4. EXAMPLES ANALYSIS

In order to verify the performance of the improved algorithm proposed in this paper, the simulation test was carried out on the actual data of the customer related activities of 4 business units of a company in 2014, as shown in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Business Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>8246</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>214.5</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>169</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>13.78</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>46</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>4827</td>
</tr>
<tr>
<td>( x_7 )</td>
<td>27</td>
</tr>
<tr>
<td>( x_8 )</td>
<td>0.5</td>
</tr>
<tr>
<td>( x_9 )</td>
<td>12.04</td>
</tr>
<tr>
<td>( x_{10} )</td>
<td>-20%</td>
</tr>
<tr>
<td>( x_{11} )</td>
<td>58%</td>
</tr>
<tr>
<td>( x_{12} )</td>
<td>0.92</td>
</tr>
<tr>
<td>( x_{13} )</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Table 2. Customer Relationship Management costs

<table>
<thead>
<tr>
<th>Variable</th>
<th>Business Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>0.89</td>
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<tr>
<td>( x_2 )</td>
<td>6.29</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>18.15</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>12.90</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>4.52</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>11.83</td>
</tr>
<tr>
<td>( x_7 )</td>
<td>6.16</td>
</tr>
<tr>
<td>( x_8 )</td>
<td>4.55</td>
</tr>
<tr>
<td>( y )</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Table 3. Input and output indicators for each business unit CRM
In Table 2, the corresponding relationships of variables are $x_i$ for sample order amount, $x_j$ for the R & D investment, $x_k$ for marketing expenditure, $x_l$ for purchase amount, $x_m$ for order management cost, $x_n$ for the product price cut loss, $x_o$ for the information work quota, $x_p$ for sales revenue, $x_q$ for sales profit rate, $x_r$ for market share, $x_s$ for customer satisfaction and $x_t$ for the customer sales revenue, respectively. According to the data of Table 2, the results were obtained shown in table 3 by using the improved model proposed in this paper and $y$ is the derived value of the output index among them. According to table 3, the company carried out the relatively effective customer relationship management on more than half of the customers of the four strategic customers, in which one was effective in pure technique and two were effective in pure scale. However, there were very huge efficiency differences among customers and the calculation results according to the model of No.1 business unit indicated that its efficiency was lower than the other customers, so its customer relationship management efficiency needs to be improved.

5. CONCLUSIONS

Customer is an important asset of the enterprise. The customer relationship has a significantly positive impact on customer loyalty. The customer satisfaction and the willingness to maintain relationships are the two dimensions of relationship which will be helpful to maintain customers and increase the purchase. Long-term relationship will promote the supply and demand sides to provide real information to share with each other, and ultimately achieve a win-win outcome. A management efficiency evaluation model was proposed based on high-dimensional index differential optimized DEA algorithm in this paper, aiming at the needs of the company’s customer management efficiency evaluation. And simulations show the effectiveness of the proposed method.

REFERENCES