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Optimal Sliding Surface Design for a MIMO Distillation System

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Abstract

This Paper presents the Sliding Mode Control of a Distillation Column System which are influenced by external disturbances or perturbations that act along the input. The issue of robustness property to be studied only for Matched Perturbation or for Matched uncertainty. The switching surface design for Sliding Mode control to be interpreted as a static feedback selection problem. The switching hyper plane design to be carried out along with pole placement technique and Quadratic Minimization technique for regulatory problems or disturbance rejection problem. The results are obtained for the distillation column where the Eigen values are to be placed arbitrarily and optimally and the robustness property to be measured using the condition number or by obtaining norm of the state variables about where the Eigen values are placed in closed Loop

Key words: Sliding Mode Control, Pole Placement Problem, Optimal Control, Matched Uncertainty, Distillation Column

1. INTRODUCTION

Sliding Mode Control is a robust control for both linear and nonlinear systems. In this paper perturbations or external bounded disturbances are to be studied only a vanishing disturbance. The vanishing disturbance is one where the disturbance vanishes about the equilibrium point. (Anusha Rani, Kumar, and Manic 2016)/Kumar and Manic 2014). By properly applying high gain feedback in the closed loop the disturbance vanishes. State feedback controller can be used to place the closed loop Eigen values. For Multi Input the design of gain is not unique for a given set of Eigen values and for the same Eigen values different gain matrix will produce different Eigen vectors and different performance. Optimal values to be found for better performance. Switching hyper plane is designed by minimising cost functions in which quadratic terms of the states are used (KAUTSKY, NICHOLS, and DOOREN 1985). This method ensures weightings placed such that the control surface follows the modes(Moore 1975). The Eigen vectors contain information about the interaction between the states which is arbitrarily fixed by pole placement design techniques. Optimal Eigen structure assignment design offers to minimise the effects of unmatched perturbations on the sliding mode dynamics optimally by designing the surface design(Edwards and Spurgeon 1988).

2. CONTROL DESIGN

2.1. Regular Form

A regular form to be used for reduced order sliding mode dynamics

Consider the system considered of the form

\[ \dot{x} = Ax + Bu \]

(1)

Where \( \text{rank} \ (B) = m \) and \((A, B)\) is a controllable pair and the switching function is represented as

\[ S(t) = Sx(t) \]

(2)

The system represented in regular form as [1]

\[ \begin{align*}
   z_1(t) &= A_{11}z_1(t) + A_{12}z_2(t) \\
   z_2(t) &= A_{21}z_1(t) + A_{22}z_2(t) + B_2u(t)
\end{align*} \]

(3a)

(3b)

And the linear sliding manifold as

\[ S(t) = S_1z_1(t) + S_2z_2(t) \]

(4)

And the change of coordinates by an orthogonal matrix \( T \)

\[ Z(t) = T^*x(t) \]

(5)

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\[ T_r = \begin{bmatrix} I_{n-m} & -B_2B_2^{-1} \\ 0 & B_2^{-1} \end{bmatrix} \]  

(6)

and change of coordinate is

Input matrix \( B \) in (2) may be partitioned (after reordering the state vector components) as

\[ B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \]  

(7)

Where \( B_1 \in \mathbb{R}^{n-m \times m} \), \( B_2 \in \mathbb{R}^{m \times n} \) with \( \det B_2 \neq 0 \)

\[ T_r A T_r^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \]  

(8)

and

\[ T_r B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \]  

(9)

And the switching function after transformation

\[ ST_r^T = [S_1 \ S_2] \]  

(10)

The switching function \( S(t) \) to be identically equal to zero during sliding motion

From Equation (4)

\[ S_1z_1(t) + S_2z_2(t) = 0. \]  

(11)

\[ z_2(t) = -Mz_1(t) \] where \( M = S_2^{-1}S_1 \) which is the existence problem to give \((n-m)\) negative poles to the closed loop system. The sliding system can become totally insensitive to matched uncertainty but it will be affected by unmatched uncertainty.

The sliding mode is governed by the above equation (11). On substituting \( z_2 \) in the first equation it becomes a closed with feedback form

\[ \dot{z}_1(t) = (A_{11} - A_{12}M)z_1(t) \]  

(12)

the techniques for switching surface selection or \( M \) to found by using pole placement technique or by using quadratic minimisation technique.

\[ ST_r = [M \ I_n] \] which is same as Equation (10)

The designed manifold can be linear or nonlinear, depends on our specified goal. The simplest manifold is a linear one.

2.1.1 Existence Condition of Sliding Mode

Objective here is to design a control input \( u \) such that, the sliding motion occurs infinite time.

\[ v = -K\text{sign}(S) \]  

(13)

where \( K > 0 \)

2.2. Pole Placement Technique

Full state feedback (FSF), or pole placement, is a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the s-plane. Placing poles is desirable because the location of the poles corresponds directly to the eigenvalues of the system, which control the characteristics of the response of the system. The system must be considered controllable in order to implement this method.

2.2.1. Principle

If the closed-loop input-output transfer function can be represented by a state space equation

\[ x = Ax + Bu \]  

(14a)

\[ y = Cx + Du \]  

(14b)

Then the poles of the system are the roots of the characteristic equation given by
Full state feedback is utilized by commanding the input vector $u$. Consider an input proportional (in the matrix sense) to the state vector, System with state feedback (closed-loop)

$$u = -Kx$$  \hspace{1cm} (16)

Substituting into the state space equations above,

$$\dot{X} = (A - BK)x$$  \hspace{1cm} (16a)

$$Y = (C - DK)x$$  \hspace{1cm} (16b)

The roots of the FSF system are given by the characteristic equation, $\text{det}[S - (A - BK)]$. Comparing the terms of this equation with those of the desired characteristic equation yields the values of the feedback matrix $K$ which force the closed-loop eigenvalues to the pole locations specified by the desired characteristic equation.

### 2.3. Linear Quadratic Regulator Method

The problem of minimising the quadratic performance index is given by

$$J = \frac{1}{2} \int_{t_0}^{\infty} x(t)^T Q x(t) \, dt$$

where $Q$ is a positive definite Identity Matrix and $t_0$ is the time at which the sliding motion starts.

$$T Q T^T = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$  \hspace{1cm} (17)

Where $Q_{22} = Q_{12}^T$, then the system is represented in the coordinate transformation as

$$J = \frac{1}{2} \int_{t_0}^{\infty} Z_1^T Q_{11} Z_1 + 2Z_1^T Q_{12} Z_2 + Z_2^T Q_{22} Z_2 \, dt$$  \hspace{1cm} (18)

$Z_1$ determines the system dynamics and the effective control input is determined by $Z_2$

$$J = \frac{1}{2} \int_{t_0}^{\infty} \left( Z_1^T (Q_{11} - Q_{12} Q_{22}^{-1} Q_{21}) Z_1 + Z_1^T Q_{22}^{-1} Q_{21} Z_1 \right) \, dt$$  \hspace{1cm} (19)

$$\dot{Q} = Q_{11} - Q_{12} Q_{22}^{-1} Q_{21}$$  \hspace{1cm} (20)

$$\dot{v} = Z_2 + Q_{22}^{-1} Q_{21} Z_1$$  \hspace{1cm} (21)

And the equation written as

$$J = \frac{1}{2} \int_{t_0}^{\infty} Z_1^T Q Z_1 + v^T Q_{22}^T v \, dt$$  \hspace{1cm} (22)

$$\dot{A} = A_1 - A_1^T Q_{22}^{-1} Q_{21}$$  \hspace{1cm} (23)

The problem of Minimization of the standard linear quadratic optimal regulator problem ensures the positive definiteness of $Q$ identity matrix which also ensures $Q_{22} > 0$, so that $Q_{22}^{-1}$ exists and also $Q > 0$.

The controllability of $(A, B)$ ensures $(A_{11}, A_{12})$ Controllable and in turn ensures $(A, A_1)$ controllable

$$Z_2 = MZ_1 = -Q_{22}^{-1} (A_{12}^T P_t + Q_{21})$$

where $P_t$ is a unique positive definite solution obtained from Algebraic Matrix Ricatti Equation.

### 3. NUMERICAL EXAMPLE

Considering the distillation column (Kautsky, Nichols, and Dooren, 1985) Where the closed loop poles has been placed for $n \times n$ matrix. In this example the matrix considered was a $5 \times 5$ matrix. By reducing it into a $3 \times 3$ matrix and also the disturbance considered where the input appears. So the system has been formulated as a reduced order system. The condition numbers given in the example taken for full order matrix which is higher rather than for a reduced order system.
\[
A = \begin{bmatrix}
-0.1094 & 0.0628 & 0 & 0 & 0 \\
1.3060 & -2.1320 & 0.9807 & 0 & 0 \\
0 & 1.5950 & -3.1490 & 1.5470 & 0 \\
0 & 0.3550 & 2.6320 & -4.2570 & 1.8550 \\
0 & 0.0023 & 0 & 0.1636 & -0.1625
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 & 0 \\
0.00638 & 0 \\
0.0838 & -0.1396 \\
0.1004 & -0.2060 \\
0.0063 & -0.0128
\end{bmatrix}
\]

Converting it into regular form where \( T \) obtained from Equation (6)

\[
TAT^{-1} = \begin{bmatrix}
-0.001 & 0.001 & 0 & 0 & 0 \\
0.0013 & 0.0184 & 0.1705 & -0.0273 & 0.0634 \\
0 & 0.0064 & 0.0369 & -0.0066 & 0.0156 \\
0 & -0.3215 & -2.6509 & 0.4267 & -1.0041 \\
0 & -0.5184 & -1.3077 & 0.2088 & 0.4917
\end{bmatrix}
\]

\[
TB = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
\]

The system is obtained as in equation (3)

\[
A_1 = \begin{bmatrix}
-0.1094 & 0.0628 & 0 \\
1.3060 & 18.3785 & 170.4915 \\
0 & 6.4216 & 36.4454
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
0.0040 & 0 \\
-27.2778 & 63.9243 \\
-6.6156 & 15.6182
\end{bmatrix}
\]

\[
A_{21} = \begin{bmatrix}
0 & -0.3215 & -2.6569 \\
0 & -0.1584 & -1.3077
\end{bmatrix}
\]

\[
A_{22} = \begin{bmatrix}
0.4297 & -1.0041 \\
0.2088 & -0.4917
\end{bmatrix}
\]

The value obtained from equation (23)

\[
A = \begin{bmatrix}
-0.1094 & 0.0631 & 0.0003 \\
1.3060 & 16.6382 & 159.2818 \\
0 & 5.9995 & 34.2107
\end{bmatrix}
\]

and \( K_{opt} = \begin{bmatrix}
271.2805 & -10.26 & 9.9633 \\
115.6243 & 3.9715 & 6.909
\end{bmatrix} \]

Thus the optimal value obtained is used for closed loop poles assignment.

4. RESULT AND DISCUSSION

The Eigen values of matrix A are [-5.23,-2.9142,-0.08576,-0.0069,-0.0785].By pole placement technique the poles are placed as [-1,-2,-3,-4,-5] and the condition number which is obtained as [13.8891, 65.3340, 15.9081, 37.9054][5].By suing optimal quadratic minimization problem M is calculated.
\[ M = \begin{bmatrix} -0.0045 & -0.0912 & -0.1734 \\ -0.0093 & 0.0562 & 0.3234 \end{bmatrix}, \]

which is a reduced order matrix and the gain \( k \) which is optimally placed for the Eigen values \( A_{11} = [0, -6.7, 62] \) and the gain computed is

\[ K_{opt} = \begin{bmatrix} 271.2805 & -10.2646 & 9.9633 \\ 115.6743 & -3.9715 & 6.9099 \end{bmatrix}. \]

In the Input Matrix \( TB = B_2 \) it has no significance on the external bounded disturbance applied to the system. It becomes invariant to the input and when the system is in sliding phase. The condition number gives what will the maximum perturbation that an Eigen value can withstand.

**Figure 1** Control Input

**Figure 2** Reduced order States

**Figure 3** Sliding Surface
5. CONCLUSION

For MIMO system considered the system has been transformed into a reduced order system and robustness properties has been obtained using sliding mode controller for matched disturbance acting along the input. The optimal gain calculated using linear quadratic method gives for the weighted values considered for an identity matrix. The Adaptive Sliding Mode controller using Fuzzy and Heuristic Algorithms for nonlinear systems has been discussed in the preceding papers (Dinesh Kumar and Meenakshipriya 2016) (Li et al. 2016). The other papers deals with only parametric robustness and not on the unmodelled dynamics which has not been considered (Vijayan and Panda, 2012a) (Vijayan and Panda, 2012b).

REFERENCES


