Co-ordinated Shipment, Ordering and Payment Policies for Deteriorating Inventory of Two Players in Supply Chain with Variable Price-Sensitive Trapezoidal Demand and Net Credit Scenario

1Nita H. Shah, 2Digeshkumar B. Shah & 3Dushyantkumar G. Patel
1Department of Mathematics, Gujarat University, Ahmedabad-380009, Gujarat, India
2Department of Mathematics, L.D. Engg. College, Ahmedabad-380015, Gujarat, India
3Department of Mathematics, Govt. Poly. for Girls, Ahmedabad-380015, Gujarat, India
1nitaashah@gmail.com, 2digeshshah2003@yahoo.co.in, 3dushyantpatel_1981@yahoo.co.in

Corresponding author: 1Nita H. Shah

Abstract - In this paper, an integrated supplier-buyer inventory model is developed when market demand is variable price-sensitive trapezoidal. The units in the inventory system of both the players are subject to deterioration at a constant rate. The supplier offers a choice between discount in unit price and credit period to do the full payment for the purchases made. This type of trade credit is known as ‘net credit’. In this scenario, if the buyer pays within permissible time $M_1$, then buyer gets discount in unit purchase price; otherwise the account must be settled by allowable time $M_2$, where $M_2 > M_1 \geq 0$. The aim is to compute optimal selling price, purchase quantity, payment time and number of transfers from the supplier to the buyer which maximizes the joint profit per unit time of the supply chain. An algorithm is proposed to determine optimal solution. The numerical example is given to validate the developed model. The managerial insights based on sensitivity analysis are highlighted.

Keywords - Supplier-buyer inventory system, deterioration, variable price-sensitive trapezoidal demand, net-credit

1 INTRODUCTION

In classical EOQ model, it is tacitly assumed that the buyer must pay immediately on purchases which is no longer practice in the business transactions. The supplier uses promotional tools to boost demand, to attract more buyers and to reduce investment tie-up. Goyal (1985) first discussed EOQ model when trade credit is offered by the supplier to the buyer. Thereafter, during last 25 years, number of analytical models are studied to obtain more intuitions into the advantages of trade credit. One can refer article by Shah et al. (2010). The settlement of payment at the later date enables the buyer to earn interest on the generated revenue.

In the review article, the facility for making early payment is not addressed. Ho et al. (2008) attested that the offer of trade credit fall outs delayed cash-flow and increase the risk of cash-flow shortage for the supplier. To shrink this trade-flow, the supplier may bid a cash discount to invite the buyer to settle payment at the prior date. For example, the supplier offers 3% discount in buyer’s unit purchase price if payment is settled within 10 days; otherwise the account is to be made by 30 days after the arrival of order quantity in buyer’s inventory system. In financial management, this type of credit term is stated as ‘3/10 net 30’. Related articles are by Lieber and Orgler (1975), Hill and Rienen (1979), Kim and Chung (1990), Arcerus and Srinivasan (1993), Ouyang et al. (2002), Chang (2002), Huang and Chung (2003) etc. and their cited references.


Deterioration is defined as the decay, change, spoilage, evaporation and loss of utility of a product from the original one. Refer to reviews on deteriorating inventory models by Nahmias (1982), Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001). Yang and Wee (2000) demarcated a heuristic algorithm to model an integrated vendor-buyer inventory model for deteriorating items. Yang and Wee (2005) discussed a win – win strategy for single-vendor single-buyer inventory supply chain when units in inventory deteriorate at a constant rate. Shah et al. (2008) protracted above model by incorporating salvage value to the deteriorated units and proved that it lowers the joint total cost.
Shah et al. (2011) modelled an integrated policy with ‘two-part’ trade credit when demand is quadratic. This type of demand is observed in seasonal products, electronic gadgets etc. It is observed that demand decreases exponentially after some time when new version of the product is floated. The demand pattern observed for any product in linearly increasing with time upto some point of time, becomes constant in some interval of time and thereafter decreases exponentially. This type of demand is stated as ‘trapezoidal demand’ (Cheng et al. (2011)). Shah and Shah (2012) modelled joint optimal inventory policy for vendor-buyer when demand is trapezoidal. They (2012) studied effect of deterioration of units on decision variables and joint total cost.

In this research, an integrated inventory system is analysed for variable price-sensitive trapezoidal demand when units deteriorate at a constant rate. The supplier offers a cash discount in unit purchase price if payment is made at pre-specified earlier date; otherwise, the buyer has to make the full payment by permissible delay period. The joint total profit per unit time is maximized with respect to payment time, retail prices in each phase of demand pattern, ordering quantity and number of shipments from the supplier to the buyer. An algorithm is proposed to find the optimal solution. A numerical example is given to validate the proposed problem. Sensitivity analysis is carried out and managerial insights are suggested.

2 ASSUMPTIONS AND NOTATIONS

2.1 ASSUMPTIONS

The mathematical model is formulated under following assumptions.

1. The supply chain comprises of Single-supplier single-buyer for single item.
2. Shortages are not allowed. Lead-time is zero.
3. The demand rate is variable price-sensitive trapezoidal. (Appendix A)
4. The supplier offers a discount $\beta$ ($0 < \beta < 1$) in unit purchase price if the buyer is willing to pay within time $M_1$; otherwise, full account is to be settled within permissible credit period $M_2$, where $M_2 > M_1 > 0$. The supplier will increase cash inflow and thereby reduce the risk of cash flow shortage by offering a discount in unit purchase price.
5. By offering a trade credit to the buyer, the supplier gets cash at a later date and hence, incurs an opportunity cost during the delivery and payment of the goods. On the other end, the buyer generates the revenue by selling the items which is deposited in an interest earning account during this allowable credit period. At the end of the credit period, buyer has to pay interest to the supplier on the unsold stock.
6. During the time $[M_1, M_2]$, a cash flexibility rate $f_{sc}$ is utilized to quantize the advantage of early cash income for the supplier.
7. The units in inventory system of both the players deteriorate at a constant rate $\theta$ ($0 < \theta < 1$). The deteriorated items can neither be repaired nor be replaced during the cycle time.

2.2 NOTATIONS

The mathematical model is formulated using following notations.

- $A_b$ Buyer’s ordering cost per order ($/order$)
- $A_s$ Supplier’s set-up cost per set up ($/set-up$)
- $I_b$ Buyer’s carrying charge fraction per unit per year excluding interest charges
- $I_s$ Supplier’s carrying charge fraction per unit per year excluding interest charges
- $\beta$ Discount rate in unit purchase price offered by the supplier to the buyer for payment within time $M_1$, to increase cash inflows which lowers the risk of cash-flow shortage
- $M_2$ Permissible time when buyer has to settle the account
  
  Note: $M_2 > M_1 > 0$
- $C_s$ Supplier’s unit production cost ($/unit$)
- $v$ Unit purchase price of the buyer ($/unit$)
- $P_1$ Buyer’s unit sale price ($/unit$) when demand is linearly increasing with time (a decision variable)
- $P_2$ Buyer’s unit sale price ($/unit$) when demand is constant with time (a decision variable)
- $P_3$ Buyer’s unit sale price ($/unit$) when demand is decreasing exponentially with time (a decision variable)
  
  Note: $P_1 < P_2 < P_3$ and $P_i > v > (1 - \beta)v > C_s$ for $i = 1, 2, 3$
- $I_{sp}$ Supplier’s capital opportunity cost rate per unit/year
Supplier’s cash flexibility rate per unit/year

Interest earned by the buyer during permissible credit period /$/year.

Interest charged by the supplier to the buyer on unsold stock per unit/year

Market demand rate (Appendix A), where \(a > 0\) is scale demand, \(0 < b_1, b_2 < 1\) are rates of change of demand, \(\eta > 1\) denotes price-elasticity and \(u_1\) and \(u_2\) are time points at which the price-sensitive demand pattern changes

The capacity utilization factor taken as the ratio of the market demand rate to production rate. \(\gamma < 1\) is deterministic and known constant

Deterioration rate \((0 < \theta < 1)\), a constant

Buyer’s cycle time in years (a decision variable)

Number of shipments from the supplier to the buyer, \(n\) is a positive integer (a decision variable)

Buyer’s procurement quantity during each shipment (a decision variable)

Buyer’s total profit per unit time

Supplier’s total profit per unit time

Joint profit of supplier-buyer per unit time

3.1 Supplier’s total profit per unit time

The supplier manufactures \(nQ\) units in batches (where \(Q\) is defined in Appendix B) and incurs a batch set-up cost \(A_s\). So the supplier’s set up cost per unit time is \(A_s / (nT)\). Using Joglekar (1988), the supplier’s inventory holding cost per unit time is given by

\[
\frac{1}{T} \left[ C_s(I_s + I_{sp}) ((n-1)(1-\gamma) + \frac{1}{T} \int_0^T I(t) dt) \right].
\]

(See Appendix C for computation of \(\int_0^T I(t) dt\).)

For each unit, the buyer’s purchase cost is \((1 - K_j\beta)\) when he pays at time \(M_j\); where \(j = 1, 2; K_1 = 1, K_2 = 0\). Hence, for the permissible credit period, the opportunity cost per unit time is \(\frac{1}{T}((1 - K_j\beta)\nu M_j Q\) where \(j = 1, 2; K_1 = 1, K_2 = 0\). When the buyer settles account at time \(M_1\), the supplier can use the revenue \((1 - \beta)\nu\) to decrease a cash flow crisis during time \(M_2 - M_1\). This early payment incurs gain at the cash flexibility rate per unit time as \(\frac{1}{T}(1 - \beta)\nu f_{sc}(M_2 - M_1) Q\).

Hence, the supplier’s total profit per unit time is

\[
TSP_j(n) = \frac{(1 - K_j\beta)\nu Q}{Tn} - \frac{C_s Q}{T} - \frac{A_s}{nT} - \frac{1}{T} \left[ C_s(I_s + I_{sp}) ((n-1)(1-\gamma) + \frac{1}{T} \int_0^T I(t) dt) \right] - \frac{(1 - K_j\beta)\nu M_j Q}{T} + \frac{(1 - \beta)\nu f_{sc}(M_2 - M_1) Q}{T}
\]

\(j = 1, 2; K_1 = 1, K_2 = 0\) (1)
3.2 Buyer’s total profit per unit time

The buyer’s ordering cost per unit time is \( \frac{A_b}{T} \), purchase cost per unit time is \( \frac{1}{T} (1 - K_j \beta) v Q \) and inventory holding cost per unit time is \( \frac{1}{T} (1 - K_j \beta) v I_b \int_0^T I(t) dt \) where \( j = 1, 2, K_1 = 1, K_2 = 0 \).

The following two cases needs to be discussed depending upon the choice of the payment time.
1. \( T < M_j \)
2. \( T \geq M_j, j = 1, 2 \).

**CASE 1: \( T < M_j \) (Fig. 1)**

In this case, the buyer’s inventory level depletes to zero before the allowable credit period. So, interest paid by the buyer is zero.

![Inventory Level](image)

**Fig. 1** Interest earned when \( T < M_j, j = 1, 2 \)

The interest earned on the generated revenue at the rate \( I_{be} \) per unit time is given in Appendix D. Hence, the buyer’s total profit per unit time is

\[
TBP_{j1}(P_1, P_2, P_3, T) = \frac{1}{T} \left[ P_1 \int_0^{u_1} R_1(P_1, t) dt + P_2 \int_0^{u_2} R_2(P_2, t) dt + P_3 \int_{u_2}^{u_3} R_3(P_3, t) dt \right] - \frac{(1 - K_j \beta) v Q}{T} \frac{A_b}{T} - \frac{(1 - K_j \beta) v I_b \int_0^T I(t) dt}{T}
\]

\[
+ \frac{I_{be}}{T} \left[ P_1 \int_0^{u_1} t \cdot R_1(P_1, t) dt + P_2 \int_0^{u_2} t \cdot R_2(P_2, t) dt + P_3 \int_{u_2}^{u_3} t \cdot R_3(P_3, t) dt \right]
\]

\[
+ \frac{I_{be}(M_2 - T)}{T} \left[ P_1 \int_0^{u_1} R_1(P_1, t) dt + P_2 \int_0^{u_2} R_2(P_2, t) dt + P_3 \int_{u_2}^{u_3} R_3(P_3, t) dt \right]
\]

\( j = 1, 2; K_1 = 1, K_2 = 0 \)
CASE 2: \( T \geq M_j; j=1,2 \) (Fig. 2)

Fig. 2 Interest earned and charged when \( T \geq M_j; j=1,2 \)

During \([0,M_j]\), the buyer sells the product and generates the revenue. On the generated revenue, buyer can earn interest at the rate \( I_{bc} \) per annum. The interest earned per unit time is

\[
\frac{1}{T} P_{1} I_{be} \left\{ \int_{0}^{M_j} t \cdot R_1(P_1,t) \, dt \right\} ; 0 \leq M_j \leq u_1
\]

\[
= \frac{1}{T} I_{be} \left\{ P_1 \left[ \int_{0}^{u_1} t \cdot R_1(P_1,t) \, dt \right] + P_2 \left[ \int_{u_1}^{M_j} t \cdot R_2(P_2,t) \, dt \right] \right\} ; u_1 \leq M_j \leq u_2
\]

\[
= \frac{1}{T} I_{be} \left\{ P_1 \left[ \int_{0}^{u_1} t \cdot R_1(P_1,t) \, dt \right] + P_2 \left[ \int_{u_1}^{u_2} t \cdot R_2(P_2,t) \, dt \right] + P_3 \left[ \int_{u_2}^{M_j} t \cdot R_2(P_3,t) \, dt \right] \right\} ; u_2 \leq M_j \leq T
\]

; where

\[ j=1,2 \]

During \([M_j,T]\), the buyer will pay interest at rate \( I_{bc} \) per annum on unsold stock. Hence, the interest charged per unit time to the buyer is

\[
\frac{1}{T} (1-K_j) I_{bc} \left\{ \int_{M_j}^{T} \left( I_j(t) dt + \int_{u_1}^{u_2} I_j(t) dt + \int_{u_2}^{T} I_j(t) dt \right) \right\} \; ; M_j \leq u_1 \leq T
\]

\[
\frac{1}{T} (1-K_j) I_{bc} \left\{ \int_{M_j}^{T} I_j(t) dt \right\} \; ; M_j \leq u_2 \leq T
\]

\[
\frac{1}{T} (1-K_j) I_{bc} \left\{ \int_{M_j}^{T} I_j(t) dt \right\} \; ; u_2 \leq M_j \leq T
\]

\[ j=1,2; K_1 = 1, K_2 = 0 \]
Therefore, the buyer’s total profit per unit time is
\[
TBP_{j2}(P_1, P_2, P_3, T) = \begin{cases} 
TBP_{j2}(P_1, P_2, P_3, T); & 0 \leq M_j \leq u_1 \\
TBP_{j2}(P_1, P_2, P_3, T); & u_1 \leq M_j \leq u_2 \\
TBP_{j2}(P_1, P_2, P_3, T); & u_2 \leq M_j \leq T \end{cases} \quad j = 1, 2
\]
(See Appendix E for \(TBP_{j2}(P_1, P_2, P_3, T); 0 \leq M_j \leq u_1 \), \(TBP_{j2}(P_1, P_2, P_3, T); u_1 \leq M_j \leq u_2 \).

The total profit of the buyer per unit time is
\[
TBP_j(P_1, P_2, P_3, T) = \begin{cases} 
TBP_j(P_1, P_2, P_3, T); & T < M_j \\
TBP_j(P_1, P_2, P_3, T); & T \geq M_j 
\end{cases} \quad j = 1, 2
\]
\[\text{(4)}\]

3.3 Joint total profit per unit time
Using (1) and (4), the supplier-buyer joint profit per unit time is given by
\[
\pi_j(n, P_1, P_2, P_3, T) = \begin{cases} 
\pi_{j1}(n, P_1, P_2, P_3, T); & T < M_j \\
\pi_{j2}(n, P_1, P_2, P_3, T); & T \geq M_j \quad j = 1, 2
\end{cases}
\]
\[\text{(5)}\]

where
\[
\pi_{j1}(n, P_1, P_2, P_3, T) = TSP_j(n) + TBP_1(P_1, P_2, P_3, T) \\
\pi_{j2}(n, P_1, P_2, P_3, T) = TSP_j(n) + TBP_2(P_1, P_2, P_3, T) \quad j = 1, 2
\]

The aim is to maximize objective function given in (5) with respect to discrete variable \(n\) and continuous variables \(P_1, P_2, P_3\) and \(T\). The following steps are performed to maximize the total joint profit per unit time of a supply chain.

4 COMPUTATIONAL STEPS
To maximize joint total profit per unit time carry out following steps.
Step 1: Assign parametric values in proper units to all model parameters.
Step 2: Set \(n = 1\).
Step 3: Solve \(\frac{\partial \pi_i}{\partial P_i} = 0\) and \(\frac{\partial \pi_j}{\partial T} = 0\), \(i = 1, 2, 3; j = 1, 2, 3\) simultaneously for \(T\) and \(P_i, i = 1, 2, 3\).
Step 4: Increment \(n\) by 1.
Step 5: Continue steps 3 and 4 until, we get
\[
\pi_j(n - 1, P_1(n - 1), P_2(n - 1), P_3(n - 1), T(n - 1)) \leq \pi_j(n, P_1, P_2, P_3, T) \geq \pi_j(n + 1, P_1(n + 1), P_2(n + 1), P_3(n + 1), T(n + 1)) \quad j = 1, 2
\]
Using optimal solution \((n, P_1, P_2, P_3, T)\), the optimal purchase quantity \(Q\) (Appendix B) per transfer for the buyer can be computed.

5 NUMERICAL EXAMPLE
Consider the following numerical values to validate the proposed problem.
\(\alpha = 90,000\), \(b_1 = 7\%\), \(b_2 = 5\%\), \(\eta = 1.25\), \(u_1 = 15\) days, \(u_2 = 45\) days, \(\gamma = 0.9\), \(C_s =$ 2/unit, \(v =$ 4.5/unit, \(A_s =$ 1000/set-up, \(A_b = $ 300/order, \(I_1 = 5\% /unit/year, \(I_2 = 8\% /unit/year, \(I_3 = 9\% /S/year, \(I_{bc} = 16\% /S/year, \(I_{be} = 12\% /S/year and \(f_{be} = 17\% /S/year and \(\theta = 12\%\). Consider, ‘3/10 net 30’ credit term thereby meaning that buyer gets 3% discount in unit purchase price if payment is made by 10 days; otherwise the payment is to be made by 30 days.

From Table 1, 14-shipments maximizes joint total profit of $ 24684 for a supply chain. The selling prices set by the buyer in corresponding demand structures are \(P_1 = $ 6.03/unit, P_2 = $ 6.27/unit and P_3 = $ 6.87/unit. The buyer orders at every 90 days and order quantity is 1513 units. For the computed optimal solution, supplier’s profit is $ 13783 and buyer’s profit is $ 10901. The concavity of total joint profit with respect to number of shipments, \(n\) and retail sale prices, \(P_1, P_2, P_3\) are shown in figures 3-6. Figures 7-9 exhibits concavity of the total joint profit for 14-shipments with respect to \((P_1, T), (P_2, T)\) and \((P_3, T)\). The variations in
permissible payment periods and its effect on decision variables and total joint profit are tabulated in Table 1. The profit gain is computed with the scenario of no credit period as \[
\left( \frac{\text{Profit with trade credit}}{\text{Profit without trade credit}} - 1 \right) \times 100%.
\]

### Table 1 Optimal Solution for various credit terms

<table>
<thead>
<tr>
<th>M1 (days)</th>
<th>M2 (days)</th>
<th>Optimal Payment Time (days)</th>
<th>n</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>T (days)</th>
<th>Q (units)</th>
<th>Profit Buyer</th>
<th>Profit Supplier</th>
<th>Profit Joint</th>
<th>Profit (%)</th>
<th>R(P,T)</th>
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<td>0</td>
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<td>16</td>
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<td>161</td>
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<td>30</td>
<td>14</td>
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<td>6.93</td>
<td>91</td>
<td>1499</td>
<td>11111</td>
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<td>14</td>
<td>5.88</td>
<td>6.06</td>
<td>6.49</td>
<td>87</td>
<td>1553</td>
<td>9328</td>
<td>15832</td>
<td>25160</td>
<td>-3.52</td>
<td>9.4</td>
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<tr>
<td>10</td>
<td>60</td>
<td>60</td>
<td>14</td>
<td>5.95</td>
<td>6.13</td>
<td>6.56</td>
<td>88</td>
<td>1545</td>
<td>10540</td>
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<td>60</td>
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<td>9.65</td>
<td>-0.94</td>
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</table>

The positive profit gain establishes that the joint decision with two-level trade credit intensive is advantageous. It also suggests that the joint profit is maximum if buyer selects the option of early payment and taking advantage of discount in unit purchase price.

![Fig. 3 Concavity of joint profit w.r.t number of shipments](image-url)
Fig. 4 Concavity of joint profit w.r.t retail price $P_1$.

Fig. 5 Concavity of joint profit w.r.t retail price $P_2$. 
Fig. 6 Concavity of joint profit w.r.t retail price $P_3$.

Fig. 7 Concavity of joint profit w.r.t retail price $P_1$ and cycle time.
Fig. 8 Concavity of joint profit w.r.t retail price $P_2$ and cycle time

Fig. 9 Concavity of joint profit w.r.t retail price $P_3$ and cycle time
In table 2, independent and joint decisions are analyzed. It is observed that the offer of trade credit decreases retail price of the buyer and the buyer is encouraged to procure larger order. The retail price of buyer in respective demand patterns are almost double in the independent decision compared to joint decision while the procurement quantity is almost halved. This results decrease in the total profit of supplier-buyer in independent decision. It is also observed that, while taking joint decision, the buyer is looser and the supplier is beneficial. To balance this, Goyal (1976) reallocated profits of the supply chain as follows.

Buyer’s profit = \[ \pi(n,P_1,P_2,P,T) \times \frac{\text{TBP}(P_1,P_2,P,T)}{\text{TBP}(P_1,P_2,P,T) + \text{TSP}(n)} \]

\[ = 24684 \times \frac{17134}{17134 + 4296} = 19736 \]

Supplier’s profit = \[ \pi(n,P_1,P_2,P,T) \times \frac{\text{TSP}(n)}{\text{TBP}(P_1,P_2,P,T) + \text{TSP}(n)} \]

\[ = 24684 \times \frac{4296}{17134 + 4296} = 4948 \]

The adjusted profits are written in the bottom line of table 2.

The inventory parameters are varied as -20%, -10%, 10% and 20% to study the changes in the joint total profit of the supply chain (fig. 10). It is seen that utilization factor and constant demand have positive impact on joint total profit. It suggests that the manufacturer should maintain production and demand ratio close to 1. The joint total profit decreases significantly with increase in buyer’s ordering cost. To combat this, the buyer should opt for larger order and thereby saving in transportation cost. The joint total profit decreases with increase in price-elasticity, supplier’s production cost, interest charged to the buyer, deterioration rate of units and supplier’s opportunity cost. One has very little control over price-elasticity and supplier’s opportunity cost but the remaining factors can be controlled by adopting proper manufacturing and storing technologies. The joint total profit increases when demand is linearly increasing with time, depicting that both the player should actively utilize this phase of demand pattern.

### Table 2 Optimal solutions for different strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Credit Term</th>
<th>Optimal Payment Time (days)</th>
<th>n</th>
<th>P_1 ($)</th>
<th>P_2 ($)</th>
<th>P_3 ($)</th>
<th>T (days)</th>
<th>R(P,T) (units)</th>
<th>Q (units)</th>
<th>Profit ($)</th>
</tr>
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<td></td>
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<td>Independent</td>
<td>Cash on delivery</td>
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<td>14.29</td>
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<td>Trade Credit</td>
<td>3/10 net 30</td>
<td>16</td>
<td>13.12</td>
<td>13.28</td>
<td>14.15</td>
<td>142</td>
<td>144</td>
<td>775</td>
<td>17134</td>
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<tr>
<td>Joint</td>
<td>Cash on delivery</td>
<td>0</td>
<td>16</td>
<td>6.24</td>
<td>6.43</td>
<td>6.73</td>
<td>86</td>
<td>161</td>
<td>1424</td>
<td>9729</td>
</tr>
<tr>
<td></td>
<td>Trade Credit</td>
<td>3/10 net 30</td>
<td>14</td>
<td>6.03</td>
<td>6.27</td>
<td>6.87</td>
<td>90</td>
<td>176</td>
<td>1513</td>
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<tr>
<td>Adjusted</td>
<td></td>
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</table>

In the adjusted profits, Buyer 19736, Supplier 4948, Joint 24684
Fig. 10 Sensitivity analysis of inventory parameters on joint profit

6 CONCLUSIONS
An integrated inventory policies are analyzed for single-supplier single-buyer when demand is price-sensitive trapezoidal. The law of demand-price is incorporated in the study. The items in inventory are subject to deterioration at a constant rate. The aim is to study the advantages of ‘net credit’ payment terms. The joint total profit is maximized with respect to number of shipments from the supplier to the buyer, payment time, retail prices for buyer in different demand phases and ordering time. The buyer is enticed to opt for joint decision by using reallocating scenario. This study motivates the buyer to do payment at earlier date and avail of discount in unit purchase price. In future, one can incorporate stochastic demand, fuzziness of inventory parameters etc.

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**Appendix A : Price-sensitive trapezoidal demand**

The trapezoidal demand \( R(P_1, P_2, P_3, t) \) is defined as

\[
R(P_1, P_2, P_3, t) = \begin{cases} 
R_0 P_1^{-\eta}; & 0 \leq t < u_1 \\
R_0 P_2^{-\eta}; & u_1 \leq t < u_2 \\
g(t) P_3^{-\eta}; & u_2 \leq t \leq T 
\end{cases}
\]

where \( u_1 \) is time point when the increasing demand function \( f(t) \) changes to constant demand and \( u_2 \) is the time point from where constant demand starts decreasing exponentially. In this study, \( f(t) \) is liner in \( t \), \( R_0 = f(u_1) = g(u_2) \) and \( g(t) \) is exponentially decreasing in \( t \). So the demand function is

\[ R(P_1, P_2, P_3, t) = R_1(P_1, t) \quad 0 \leq t < u_1 \\
R_1(P_1, t) = a(1 + bP_1)P_1^{-\eta} \]

where

\[ R_2(P_2, t) = a(1 + bP_2)P_2^{-\eta} \]

\[ R_3(P_3, t) = a(1 + bP_3)e^{-b_2(t - u_2)} \]

Appendix B: Computation of inventory at any instant of time \( t \) and purchase quantity \( Q \)

The rate of change of inventory is due to variable price-sensitive trapezoidal demand and deterioration of units. It is governed by the differential equation

\[ \frac{dI(t)}{dt} = -R(P_1, P_2, P_3, t) - \partial I(t); 0 \leq t \leq T \]

with the initial condition: \( I(T) = 0 \).

The solution of the differential equation is

\[ I(t) = \begin{cases} I_1(t); & 0 \leq t \leq u_1 \\
I_2(t); & u_1 \leq t \leq u_2 \\
I_3(t); & u_2 \leq t \leq T \end{cases} \]

where

\[ I_1(t) = aP_1^{-\eta} \left[ -\frac{1 + bP_1}{\theta} + \frac{1 + bP_1}{\theta} e^{\frac{\theta}{\theta} - \theta} - \frac{bP_1}{\theta^2} e^{\frac{\theta}{\theta} - \theta} \right] + P_2^{-\eta} \left[ a(1 + bP_2)e^{\frac{\theta}{\theta} - \theta} \right] \\
I_2(t) = aP_2^{-\eta} \left[ (1 + bP_2) \right] \left[ -1 + e^{\frac{\theta}{\theta} - \theta} \right] + P_3^{-\eta} \left[ a(1 + bP_3)e^{\frac{\theta}{\theta} - \theta} \right] \\
I_3(t) = P_3^{-\eta} \left[ a(1 + bP_3) \right] \left[ -e^{\frac{\theta}{\theta} - \theta} \right] \left[ e^{-b_2(t - u_2) - \theta} \right] \\
\]

Initial purchase quantity is given by

\[ Q = I(0) = aP_1^{-\eta} \left[ -\frac{1}{\theta} + \frac{1 + bP_1}{\theta} e^{\frac{\theta}{\theta} - \theta} - \frac{bP_1}{\theta^2} e^{\frac{\theta}{\theta} - \theta} \right] + P_2^{-\eta} \left[ a(1 + bP_2)e^{\frac{\theta}{\theta} - \theta} \right] \\
+ P_3^{-\eta} \left[ a(1 + bP_3)e^{\frac{\theta}{\theta} - \theta} \right] \left[ e^{-b_2(t - u_2) - \theta} \right] \\
\]

Appendix C: Computation of total inventory during \([0, T]\)

Total inventory during \([0, T]\) is given by

\[ T \int_{0}^{T} I(t) dt = \int_{0}^{u_1} I_1(t) dt + \int_{u_1}^{u_2} I_2(t) dt + \int_{u_2}^{T} I_3(t) dt \]

Appendix D: Computation of total interest earned

\[ \frac{P_1I_{bc}}{T} \int_{0}^{T} t \cdot R_1(P_1, t) dt + \frac{P_2I_{bc}}{T} \int_{u_1}^{u_2} t \cdot R_2(P_2, t) dt + \frac{P_3I_{bc}}{T} \int_{u_2}^{T} t \cdot R_3(P_3, t) dt \]

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