Parameter Estimation for Defocus Blurred Image Based on Polar Transformation

Min Liang
School of Information Management, Shanxi University of Finance and Economics, Taiyuan030031, Shanxi, China

Abstract
The accurate estimation scheme of point spread function (PSF) is very important for obtaining good restoration performance in image restoration. For the unknown blur parameter of a single defocus image, an estimation method is proposed. First, polar transformation is applied to the logarithmic power spectrum of the defocus image in the frequency domain. Then the defocus blur radius can be obtained by calculating the sum of the logarithmic spectrum along concentric circles in the polar coordinate. The experiment results show that for noise free blurred images, the method can achieve more accurate estimation of the point spread function within the blur radius range of $0 < R \leq 55$ pixels; for Gaussian noise blurred images, the restoration of blurred and noisy images demonstrates good performance.

Key words: Parameter Estimation, Defocus Blur, Point Spread Function, Polar Transformation

1. INTRODUCTION
Defocus blur that appears in images may be caused by defocusing of the lens. The restoration of blurred images depends on the blurring system model. Mathematically, defocus blur image is usually modeled as a convolution of point spread function (PSF) with the clear image represented by its intensities. Often, the defocus PSF is characterized by one parameter, namely defocus radius. Various methods for the estimation of blur parameter have been proposed in literature (Lin, Zhang and Shi, 2004; Lee, Fathi and Song, 2010; Zhu, Cohen, Schiller et al., 2013).

Defocus blur is estimated using line spread function (LSF) information, and then the iterative technique is used to achieve desired results (Bhaskar, Hite and Pitts, 1994). However, this method works only for limited areas of frequency. Wu, Shiqian al. (Wu, Lin, Jian, et al., 2005) presented a method which analyzes LSF to find the exact location of blur edges in spatial domain and then used this information for defocus parameter estimation. But in presence of noise, it is difficult to find exact location of edges.

Most of researchers concentrated on the determination of point spread function parameters in the spectral domain. Cannon has identified the point spread function parameters in the power spectrum of the image by inspecting the negative peak (Cannon, 1976). Vivirito et al. used bayer patterns analysis to extract edge detection details and used this information to find defocus blur (Vivirito, Battato, Curti, et al., 2002). Moghaddam presented an iterative algorithm using optical transfer function estimate blur parameter (Moghadam, 2007; Moghaddam, 2008). However, this method is noise independent; it requires manually adjustment of some parameters. Some other methods presented in (Jiang, Wu, Guo, 2005; Su, Li, Xu, et al., 2008; Chen and Yen, 2012) have used wavelet coefficients as features to train and test the radial basis function or cellular neural network for parameter estimation.

In this paper, we proposed a method for blur estimation from a single image. The polar transformation is utilized on the spectrum of the blurred image in the frequency domain. Our method was tested on standard images that were degraded with different parameters, with additive Gaussian noise, and rather precise results were shown.

2. IMAGE DEGRADATION MODEL
Generally, the image degradation process in spatial domain can be modeled by the following convolution process:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y)$$

(1)

Where $g(x, y)$ is the degraded image, $f(x, y)$ is the uncorrupted original image, $h(x, y)$ is the point spread function (PSF) that caused the degradation and $n(x, y)$ is the additive noise. Since, convolution in spatial domain $(x, y)$ is equivalent to multiplication in frequency domain $(u, v)$, (1) can be written as

$$G(u, v) = F(u, v)H(u, v) + N(u, v)$$

(2)

In most cases, the out of defocus blur caused by a system with circular aperture can be modeled as a uniform disk with radius $R$ given by
The frequency response of (3) is given by (4) which is based on a Bessel function of the first kind

\[ H(\mu, \nu) = 2\pi R J_1(R\sqrt{\mu^2 + \nu^2}) \sqrt{\mu^2 + \nu^2} \]  

(4)

Where \( J_1(\cdot) \) is the one-order Bessel functions of first kind and \( R \) is radius of uniform disk. \( H(\mu, \nu) \) is circle symmetry, and the first frequency domain nulls’s trace of the degraded image is circular. If the radius of inner circle is \( d \), and the Fourier transform’s dimension is \( L \times L \), there exist (Li, Wang and Zhao, 2009):

\[ R = \frac{3.83L}{2\pi d} \]  

(5)

Figure 1 shows the lenna image and corresponding Fourier spectrums of defocus blur images with specified radius. It could be found the alternating light and dark concentric circle stripes in the frequency spectrum of blurred images. The distance between two dark circular stripes decreases when the defocus blur radius \( R \) increases.

(a)  (b)  (c)

(d)  (e)

**Figure 1.** Defocus blurred images and their spectrums. (a) Image of size 256×256, (b) Image degraded by defocus blur using \( R=5 \) pixels, (c) Frequency response of (b), (d) Image degraded by defocus blur using \( R=10 \) pixels, (e) Frequency response of (d)

3. PSF PARAMETER ESTIMATION METHODOLOGY

According to the above mentioned analysis, to determine the blur radius \( R \), we can identify dark concentric circular stripes in the frequency spectrum of blurred images. So, the spectrum is treated as an image. When the center of corresponding spectrum image of defocus blur is treated as a pole in polar coordinate, \( S(r, \theta) \) is the spectral function of the defocus blurred image, \( r, \theta \) are variables in polar coordinate system. When the sum of \( \theta \) from 0 to 360 degrees is calculated,

\[ S(r) = \sum_{\theta=0}^{360} S_\theta(r) \]  

(6)

\( S(r) \) shows the spectral properties of a circle centered at the pole with polarradius \( r \). When \( r \) changes within a certain range, spectral amplitude curve \( S(r) \) of blurred image can be obtained under different polar radius \( r \).

Due to the range of \( S(r) \) is too wide, We use the following logarithmic transformation to gain the significant fluctuation curve effect.
So, periodic spectrum dark ring necessarily reflect the position of the minimum value. The first zero-crossings ring with radius $d_{r}$ corresponds to the minimum value of the closest to the original position.

According to (5), we can obtain the defocus blur radius $R$.

Figure 2 shows $\log S(r)$ curves of spectrum images of defocus blurred images with special radius, and it could be found that periodic minimum values of the curves correspond to the dark circular stripes.

**Figure 2.** $\log (S(r))$ curves of Figure 1(c) and Figure 1(e). (a) $\log (S(r))$ curves of Figure 1(c), (b) $\log (S(r))$ curves of Figure 1(e)

So, the algorithm for finding radius $R$ can be briefly summarized into the following steps:

**Step1:** Compute the Fourier transform $G(u,v)$ of the blurred image $g(x,y)$.

**Step2:** Calculate the logarithmic spectrum of $G(u,v)$.

**Step3:** Take the center of the logarithmic spectrum as the origin of the polar coordinate, and then get the spectral function $S(r,\theta)$. Here, $\theta$ is in $[0,2\pi]$, $r$ is in $[2,\min (M/2, N/2)-1]$, $[M, N]$ is the size of $g(x,y)$.

**Step4:** Calculate $S(r)$ according to (6), find the minimum value $r^* = \arg \min log(S(r))$ of the first zero-crossings ring.

**Step5:** According to (5) to get defocus blur radius $R$.

4. EXPERIMENTAL RESULTS

The experiments are performed on the standard images of size 256×256 (like lenna, camera, etc) which were degraded by different values of blur radius.

The experimental results with blurred images are shown in Fig. 3 and Fig. 4.

Fig. 3 shows the blurred image of camera with $R=5,10,20,40$.

Fig. 4 shows the $\log S(r)$ curves of the blurred images from Fig. 3. It is easy to find periodic ridges. The red arrow indicates the minimum point location which is close to the origin.

**Figure 3.** Camera defocus blurred images. (a) Defocus Blurred image with R=5, (b) Defocus Blurred image with R=10, (c) Defocus Blurred image with R=20, (d) Defocus Blurred image with R=40
Figure 4. LogS(r) curves of Figure 3. (a) LogS(r) curve of Figure 3(a), (b) LogS(r) curve of Figure 3(b), (c) LogS(r) curve of Figure 3(c), (d) LogS(r) curve of Figure 3(d).

Table 1 shows the performance comparison of estimating defocus parameters from degraded images without noise. When increasing the value of blur radius R, the cepstrum of blurred images decay rapidly (Zhou, Yan, Wang, 2007; Wu, Lu, et al., 2007). It is obvious that our detected blur radius gives better precision for blurred images using the mentioned real radius when $0 < R \leq 55$ pixels.

<table>
<thead>
<tr>
<th>Detected method comparison</th>
<th>Real radius of defocus blurred images</th>
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<tbody>
<tr>
<td></td>
<td>2  5  10  15  20  30  40  50  55  57</td>
</tr>
<tr>
<td>Cepstrum method</td>
<td>0.5  5  9  14  18.5  29  59.5  48.5  50  23</td>
</tr>
<tr>
<td>Our method</td>
<td>2.0  5.0  9.8  15.6  19.5  31.2  39  52  52  31.2</td>
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Fig. 5 shows the restoration results by method (Zhou, Yan, Wang, 2007) and our proposed method when $\text{nsr}=0.01$. The wiener filter algorithm was performed to restore the original image after the blur parameters were calculated. It is clear that the restoration result of the proposed algorithm is better and the details can be recognized very well.
5. CONCLUSIONS

In the paper, we presented a novel algorithm of parameter estimation of defocus blurred image restoration. Using polar transformation on the logarithmic spectrum of a defocus blurred image, the spectrum characteristic of concentric circles is strengthened. The experiment results show that the algorithm is proved to be effective for finding the radius of defocus blurred images and robust in noisy environment. The restored image quality is improved obviously.

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REFERENCES


