Petri Nets Model Based on Matrix Operation

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Abstract
In view of the issue of the single membership grade that exists in the fuzzy Petri nets (FPN), with the combination of the matrix computation and Petri nets theory, the Matrix Operation Petri Nets (MOPN) model is constructed and applied for the representation and reasoning of the knowledge. Firstly the MOPN model is constructed, and applied to the representation of the knowledge, and through the introduction of the inhibition transition arc in the model, the issue of the representation of the negative proposition is solved. Secondly, the MOPN reasoning algorithm is put forward, and by the modification of the transfer rules of the token value after the transitioning and triggering, it has solved the issue of the retention of the fact in the process of reasoning. Finally, the reasoning algorithm is analyzed, and examples are provided to validate the feasibility of the proposed MOPN model and its inference algorithm, the results show that the MOPN is an effective expansion and development of FPN, and its description of the reasoning result is more delicate and comprehensive.

Keywords: Intuitionistic Fuzzy Petri Nets, Intuitionistic Fuzzy Production Rule, Knowledge Representation, Intuitionistic Fuzzy Reasoning.

1. INTRODUCTION
FPN is a good modeling tool of the knowledge system that is based on the Fuzzy Production Rules (FPRs), since FPN was put forward, scholars at home and abroad have carried out thorough and in-depth research on the knowledge representation and reasoning method that is based on FPN (Konopacki, 2005; Kawase, Newsome and Inoue, 2002; Li, Liu, Shao and Yin, 1995). The representation and reasoning algorithm of the knowledge that is based on FPN introduces the weight value into the FPN, and puts forward the Weighted Fuzzy Petri Nets (WFPN) (Kramers and Wanner, 1941). The representation method of the negative propositions in the Fuzzy Reasoning Petri Nets (FRPNs) puts forward the FRPNs inference algorithm that is based on the matrix computation (Yokoi, 1996). The representation method of the negative proposition has the inconsistency problem, and the Consistent Fuzzy Petri Nets (CFPN) containing the negative propositional logic applies the same places to represent the original proposition and the negative proposition at the same time (Yao and Zhao, 2009). The FPN formal reasoning algorithm that is based on the matrix operation has made full use of the parallel computing capacity of the Petri nets, however, the credibility of the computed result proposition could be greater than 1 (Konerz, Dohmen, Liu, Beholz, Dushe, Posner, Lembeke and Erdbrugger, 2005). With the increasingly complication of the knowledge representation, the traditional FPN cannot meet the demand of the knowledge representation very well. And many scholars have made extension on the traditional FPN, and put forward a variety of extended FPN modes. The Adaptive Fuzzy Petri Nets that has the learning ability (AFPN) (Dongarra, Sameh and Sorensen, 1986), and the Dynamic Adaptive Fuzzy Petri Nets (DAFPN) (McKellara and Coffman, 1969) model has the dynamic adaptive capability, therefore can more accurately represent the complex expert system that is based on the knowledge. High - Level Fuzzy Petri Nets (HLFPN) can represent the IF - THEN and the IF - THEN - ELSE rules at the same time, and thus has the ability to solve the issue of the negative proposition (McCulloch, 2007; Kajani, Venecheh and Ghasemi, 2009).

In recent years, FPN has been widely used in the knowledge representation and reasoning, modeling simulation, fault diagnosis and other fields, but the study on FPN has always been confined to the combination of the Zadeh Fuzzy Sets (ZFS) theory and Petri Nets theory (Challacombe, 2000; Siek and Lumsdaine, 1998). ZFS applies a single scale (that is, the membership grade or membership grade function) to define the fuzzy sets, which can only describe the “Fuzzy Concept” of "There is no difference between one aspect and the other", while cannot represent the neutral state, when FPN inherits the advantages of ZFS, it has also inherited its defects of the single membership grade at the same time. While in Intuitionistic Fuzzy Sets (IFS), due to the addition of a new attribute parameter—the non-membership grade functions, it can describe the concept of "neutrality", which, therefore, is more exquisite and thorough in the depiction of the fuzzy nature of the objective world, and hence is one of the most influential expansion and development for ZFS (Huber and Borodjevic, 1995; Fischer, Kemmer, Gklein, Locker, Lutz, Neeser, Struder and Wermes, 2000). Therefore, this paper applies the MOPN model constructed by the combination of the matrix theory and the Petri nets theory for the knowledge representation and reasoning. Through the introduction of the inhibition transition arc into the MOPN model, the transfer rules of the token value of the places after the modification of the transitioning and...
triggering as well as the triggering conditions of the transition have solved the representation of the negative proposition, the retention of the facts, the repetitive triggering of transitions and other problems.

2. KNOWLEDGE REPRESENTATION BASED ON THE MOPN MODEL

2.1. Representation of the Negative Proposition

In a rule set, the original proposition and the negative proposition of the proposition may exist at the same time. For example, in the rule set $S_1 = \{R_1, R_2\}$, in which

$$R_1: If d_i \land \neg d, THEN \neg d_4$$
$$R_2: If d_i \land \neg d, THEN d_4 \land \neg d_5$$

To reasonably represent the original proposition and the negative proposition, the literature represents the native meaning by the introduction of the negative weight values; two different places are applied by the literature to represent the original proposition and the negative proposition respectively, which, however, will increase the complexity of the model; the literature adopts the inhibition arcs and new places to represent the negative proposition in the premise condition and the conclusion respectively, but there is the problems that the representation of the model is not unique and the reasoning results are in conflicts, etc. According to the understanding of the literature, the original proposition and the negative proposition play the facilitation and inhibition role in the rule on the proposition respectively (For example, in $R_1$, $\neg d_2$ has hampered the occurrence of $d_4$ in the reasoning), through adding the signage “−” on the transfer arc of the model to distinguish the facilitation and inhibition role (The unmarked transfer arc is the positive transfer arc, which indicates the facilitation effect; the transfer arc with “−” is the inhibition transfer arc, which indicates the inhibition effect), and the original proposition and the negative proposition are successfully represented with the same places at the same time, which has avoided changing the original structure of the model.

This paper introduces this representation method into the MOPN model, so as to represent the original proposition and the negative proposition. The rule set $S_1$ can be demonstrated in Figure 1.

![Figure 1. MOPN Model of Rule Set S1](image)

2.2. Knowledge Representation Method Based on MOPN

The corresponding relationship between the IFPR set and MOPN model can be represented in the following, as shown in Table 1.

<table>
<thead>
<tr>
<th>IFPR Set</th>
<th>MOPN Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inference Rule $R_j$</td>
<td>Transition $t_j$</td>
</tr>
<tr>
<td>The Premise of $R_j$</td>
<td>Input Places of $t_j$</td>
</tr>
<tr>
<td>Conclusion of $R_j$</td>
<td>Output Places of $t_j$</td>
</tr>
<tr>
<td>Proposition $d_j$</td>
<td>Places $p_i$</td>
</tr>
<tr>
<td>True Value of $d_j$</td>
<td>Token Value of $p_i$</td>
</tr>
<tr>
<td>Threshold Value of $R_j$</td>
<td>Threshold Value of $t_j$</td>
</tr>
<tr>
<td>Credibility of $R_j$</td>
<td>Credibility of $t_j$</td>
</tr>
<tr>
<td>Application of $R_j$</td>
<td>Triggering of $t_j$</td>
</tr>
</tbody>
</table>

According to the corresponding relations as shown in Table 1, the IFPR set can be mapped as MOPN.
(1) Simple IFPRMOPN Model

In which, places $p_j$ and $p_k$ represent the prerequisite proposition and the conclusion proposition of the rule respectively; and the token value $\theta_j = (\mu_j, \gamma_j)$ and $\theta_k = (\mu_k, \gamma_k)$ respectively and the true value of $d_j$ and $d_k$; $\lambda_i = (\alpha_i, \beta_i)$ represents the threshold value of the transition $t_i$ (that is, the threshold value of rule $R_i$); $CF_i = (C\mu_i, C\gamma_i)$ represents the credibility of transition $t_i$ (that is, the credibility of rule $R_i$), and $\theta_j$, $\theta_k$, $\lambda_i$, and $CF_i$ are intuitionistic fuzzy numbers.

If and only if $\mu_j \geq \alpha_i, \gamma_j \leq \beta_i$ is met at the same time, can the transition $t_i$ be triggered (that is, the rule $R_i$ is applied), the token value of $p_k$ (that is, the true value of $d_k$) $\theta_k = (\mu_k, \gamma_k)$, in which:

$$\left\{ \begin{array}{l}
\mu_k = \mu_j \times C\mu_i \\
\gamma_k = \gamma_j + C\gamma_i - \gamma_i \times C\gamma_i
\end{array} \right.$$  (1)

(2) IFPR MOPN Model with the Conjunction Form of Premise Condition

Assume the true value of $d_m (m = 1, 2, \cdots, n)$ and $d_k$ are $\theta_j = (\mu_j, \gamma_j)$ and $\theta_k = (\mu_k, \gamma_k)$, $\lambda_i = (\alpha_i, \beta_i)$, $CF_i = (C\mu_i, C\gamma_i)$ respectively.

If and only if $\min (\mu_j, \mu_j, \cdots, \mu_m) \geq \alpha_i$ and $\max (\gamma_j, \gamma_j, \cdots, \gamma_m) \leq \beta_i$ are met at the same time, can $t_i$ be triggered (that is, $R_i$ is applied), the token value of $p_k$ (that is, the true value of $d_k$) $\theta_k = (\mu_k, \gamma_k)$, in which:

$$\left\{ \begin{array}{l}
\mu_k = \min (\mu_j, \mu_j, \cdots, \mu_m) \times C\mu_i \\
\gamma_k = \max (\gamma_j, \gamma_j, \cdots, \gamma_m) + C\gamma_i - \max (\gamma_j, \gamma_j, \cdots, \gamma_m) \times C\gamma_i
\end{array} \right.$$  (2)

(3) IFPR MOPN Model with the Disjunction Form of Premise Condition

Assume the true value of $d_m (m = 1, 2, \cdots, n)$ and $d_k$ are $\theta_j = (\mu_j, \gamma_j)$ and $\theta_k = (\mu_k, \gamma_k)$, $\lambda_i = (\alpha_i, \beta_i)$, $CF_i = (C\mu_i, C\gamma_i)$ respectively.
Assume the true value of \( d_{jm} (m=1,2,\cdots,n) \) and \( d_k \) are respectively \( \theta_{jm} = (\mu_{jm}, \gamma_{jm}) \), \( \theta_k = (\mu_k, \gamma_k) \), \( \lambda_i = (\alpha, \beta) \), \( CF_i = (C \mu_i, C \gamma_i) \).

If there are \( m \) places \( p_{j1}, p_{j2}, \cdots, p_{jm} \) \( (1 \leq m \leq n) \) in \( p_{j1}, p_{j2}, \cdots, p_{jm} \) with the token values that meet \( \mu_{jm} \geq \alpha, \gamma_{jm} \leq \beta \) at the same time, then the transition \( t_{j1}, t_{j2}, \cdots, t_{jm} \) is triggered (at this point \( R_i \) is applied), the token value of \( p_i \) (that is, the true value of \( d_k \)) \( \theta_k = (\mu_k, \gamma_k) \), in which :

\[
\begin{align*}
\mu_k &= \max \left( \mu_{j1} \times C \mu_i, \mu_{j2} \times C \mu_i, \cdots, \mu_{jm} \times C \mu_i \right) \\
\gamma_k &= \min \left( \gamma_{j1} + C \gamma_i - \gamma_{j1} \times C \gamma_i, \gamma_{j2} + C \gamma_i - \gamma_{j2} \times C \gamma_i, \cdots, \gamma_{jm} + C \gamma_i - \gamma_{jm} \times C \gamma_i \right)
\end{align*}
\]

(3) IFPR MOPN Model with the Conjunctive Form of Conclusion

Assume the token values of \( p_{j1}, p_{j2}, \cdots, p_{jm} \) are \( \theta_{j1}, \theta_{j2}, \cdots, \theta_{jm} \) respectively, in which \( \theta_j = (\mu_j, \gamma_j) \), \( \theta_k = (\mu_k, \gamma_k) \), \( i = 1,2,\cdots,n \), \( \lambda_i = (\alpha, \beta) \), \( CF_i = (C \mu_i, C \gamma_i) \).

If and only if \( \mu_j \geq \alpha, \gamma_j \leq \beta \) is met at the same time, can \( t_i \) be triggered (that is, \( R_i \) is applied), the token value of \( p_k \) (that is, the true value of \( d_k \)) \( \theta_k = (\mu_k, \gamma_k) \), in which :

\[
\begin{align*}
\mu_k &= \mu_j \times C \mu_i \\
\gamma_k &= \gamma_j + C \gamma_i - \gamma_j \times C \gamma_i
\end{align*}
\]

(4) IFPR MOPN Model with the Conjunctive Form of Conclusion

3. REASONING ALGORITHM BASED ON MOPN

3.1. Reasoning Algorithm Based on MOPN

Acyclic network refers to the network without loop or circular form, and in most practical applications the knowledge base almost has no existing loop. Therefore, this paper assumes that the established MOPN model based on IFPR is an acyclic network, that is, there is no loop existing in the model.

Definition 4 (immediate reachability set, reachability set) in the MOPN model, assume \( t_i \) is a transition, \( p_i, p_j \) and \( p_k \) are places, if \( p_i \in I(t_i) \cap IN(t_i) \) and \( p_j \in O(t_i) \cap ON(t_i) \), then it can be called that it is immediately reachable from \( p_i \) to \( p_j \). If \( p_j \) is immediately reachable from \( p_i \) to \( p_j \), and \( p_j \) is also immediately reachable from \( p_i \) to \( p_k \), then it can be called that it is reachable from \( p_i \) to \( p_k \). The set composed of all the places with immediate reachability from \( p_i \) is called the immediate reachability sets of \( p_i \), which is recorded as \( IRS(p_i) \). And the set composed of all the reachability sets from \( p_i \) is called the reachability set of \( p_i \), which is recorded as \( RS(p_i) \).

Suppose there are \( n \) propositions and \( m \) rules in IFPR set \( S \), there are \( n \) places and \( m \) transitions in the corresponding MOPN model, then the reasoning algorithm based on MOPN is as follows:

Algorithm 1 Reasoning Algorithm Based on MOPN

Input : Input positive transfer matrix \( I \), input inhibition transfer matrix \( IN \), output positive transfer matrix \( O \), output inhibition transfer \( ON \), the transition threshold value \( Th \), the credibility of the rule \( CF \), the initial true value of the proposition \( \theta^0 \).

Output : The token value of the places (that is, the true value of the proposition), the number of iterations \( k \).
Pretreatment (Make judgment on whether there is loop existing in the MOPN model): In the MOPN model, \( \exists p_i \in RS(p_i) \), then there is loop existing in the model, hence the model cannot apply the reasoning algorithm, exit.

Step1 Initialization of all the inputs, let \( k = 1 \), \( \vartheta^{-1} = \vartheta = (\vartheta^1, \vartheta^2, \ldots, \vartheta^n)^T = (\vartheta_1, \vartheta_2, \ldots, \vartheta_n)^T \), the true values of the unknown propositions are represented with \( (0, 1) \), and the initial equivalent input \( \rho_{k-1} = \rho_0 = ((0, 1), (0, 1), \ldots, (0, 1))^T \).

Step2 Calculate the equivalent inputs of each transition, that is, make all the token values input in the places of each transition equivalent to the token value of single input places, and the results are as the following

\[ \rho_k = (\rho \vartheta_1, \rho \vartheta_2, \ldots, \rho \vartheta_n)^T = ((\rho \mu_1, \rho \gamma_1), (\rho \mu_2, \rho \gamma_2), \ldots, (\rho \mu_n, \rho \gamma_n))^T \]

In which, \( \rho \mu_i = \min \left\{ x_i \left| x_i \right. \right\} \] in the places of each transition equivalent to the token value of single input places, and the results are as the following

\[ \rho \gamma_j = \max \left\{ y_i \left| y_i \right. \right\} \]

That is

\[ \rho_k = (I^T \odot \vartheta_{k-1}) \oplus (IN^T \odot \vartheta_{k-1}) \quad (5) \]

Step3 (Repetitive triggering of inhibition transition) Determine whether the equivalent input of each transition is greater than the former input, at this point what \( \rho \vartheta_k \) records is the equivalent input of the transition that is necessary to be triggered. If \( \rho \vartheta_k = ((0, 1), (0, 1), \ldots, (0, 1))^T \), then the reasoning ends, and output proposition end value \( \vartheta^k \), at this point \( \vartheta^k = \vartheta^{k-1} \); Or else, the reasoning shall continue.

Step4 Compare the transition equivalent input with the rule threshold value, and keep the inputs that can trigger the transition;

Step5 After the transition is triggered, the computation results of the true value of the proposition are as follows:

\( 1 \) \( S_i = CF \square \rho \vartheta^k \), in which

\[ S_i = (s_1^i, s_2^i, \ldots, s_n^i)^T = ((s \mu_1^i, s \gamma_1^i), (s \mu_2^i, s \gamma_2^i), \ldots, (s \mu_n^i, s \gamma_n^i))^T \]

\[ s \mu_i^i = C \mu_i^i \times \rho \mu_i^i \]

\[ s \gamma_j^i = C \gamma_j^i + \rho \gamma_j^i - C \gamma_j^i \times \rho \gamma_j^i \]

\( 2 \) \( Y_i = (y \vartheta_1^i, y \vartheta_2^i, \ldots, y \vartheta_n^i)^T = ((y \mu_1^i, y \gamma_1^i), (y \mu_2^i, y \gamma_2^i), \ldots, (y \mu_n^i, y \gamma_n^i))^T \)

In which

\[ y \mu_i^i = \max \left\{ x_i \left| x_i \right. \right\} \]

\[ y \gamma_j^i = \min \left\{ y_i \left| y_i \right. \right\} \]

That is

\[ Y_i = (O \odot S_i) \odot (ON \odot \overline{S_i}) = [(O \ominus (CF \square \rho \vartheta^i)) \odot \overline{ON \odot (CF \square \rho \vartheta^i)}] \quad (6) \]

Step6 Calculate the token values of all the places (that is, the end true values of all propositions):
\[ \theta^k = \theta^{k-1} \oplus Y_k = (\theta^k_1, \theta^k_2, \ldots, \theta^k_n)^T = (\mu^k_1, \gamma^k_1, \mu^k_2, \gamma^k_2, \ldots, \mu^k_n, \gamma^k_n)^T \] (7)

This step retains the fact.

Step 7 Determine whether the reasoning is ended:
If \( \theta^k = \theta^{k-1} \), the reasoning ends, output proposition end value \( \theta^k \); otherwise, let \( k = k + 1 \), go to Step 2.

3.2. Algorithm Analysis

Define 5 (Source places, sink places)
If a place does not have the input places, this place is called Source Places; if a place does not have the output place, this place is called Sink Places.
Define 6 (Route)
For a given place \( p \), if \( p \) can acquire the token value from the source places through the order of transition \( t_1, t_2, \ldots, t_n \), it is called that the transition sequence \( t_1, t_2, \ldots, t_n \) is a route of the places \( p \). If the transition sequence \( t_1, t_2, \ldots, t_n \) can be triggered successively, then this route can be called the active route.

Theorem 2 \( \rho^k_t = ((0,1),(0,1),\ldots,(0,1))^T \) is the sufficient condition for the end of the reasoning, but not the necessary condition.

Prove (1) When \( \rho^k_t = ((0,1),(0,1),\ldots,(0,1))^T \), it can be known from Equation (6~7) that, \( \rho^k_t = (0,1), (0,1), \ldots, (0,1))^T \), \( Y_k = ((0,1),(0,1),\ldots,(0,1))^T \), \( \theta^k = \theta^{k-1} \oplus Y_k = \theta^{k-1} \), at this point, the reasoning must end, that is, \( \rho^k_t = ((0,1),(0,1),\ldots,(0,1))^T \) is the sufficient condition for the end of the reasoning.

(2) When \( \rho^k_t = ((0,1),(0,1),\ldots,(0,1))^T \), if the equivalent input of the transition is not greater than the threshold value, then \( \rho^k_t = \rho^{h-1}_t \), \( \theta^k = \rho^{h-1} \oplus Y_k = \theta^{h-1} \). Therefore, \( \rho^k_t = ((0,1),(0,1),\ldots,(0,1))^T \) is not the necessary condition for the end of the reasoning.

Theorem 3 \( \theta^k = \theta^{k-1} \) is the necessary and sufficient condition for the end of the reasoning.

This theorem is obviously established and the proving is skipped.

Theorem 4 The reasoning algorithm can be terminated after \( k \) times of infinite loops, in which \( 1 \leq k \leq h + 1 \), and \( h \) represents the number of transitions of the longest route in the MOPN model.

Prove (1) Firstly prove that the reasoning algorithm can be terminated after \( k \) times of infinite loops.
Assume that it ends at the \( k \) times of reasoning, \( \theta^k = \theta^{k-1} \), apparently, according to the Theorem 3, the reasoning has ended.

(2) Then further prove \( 1 \leq k \leq h + 1 \)
(i) Firstly prove \( k = h + 1 \)
Assume \( h \) represents the number of transitions of the longest route in the MOPN model, we only need to prove that when \( k = h + 1 \), after the reasoning ends \( \rho_{k+1} = ((0,1),(0,1),\ldots,(0,1))^T \) or \( \theta^{k+1} = \theta^k \).

It is known that \( \theta^k = (\theta^k_1, \theta^k_2, \ldots, \theta^k_n)^T = (\mu^k_1, \gamma^k_1, \mu^k_2, \gamma^k_2, \ldots, \mu^k_n, \gamma^k_n)^T = (\mu^{h+1}_1, \gamma^{h+1}_1, \mu^{h+1}_2, \gamma^{h+1}_2, \ldots, \mu^{h+1}_n, \gamma^{h+1}_n)^T \).

Assume that \( p_j \) is the sink places of the longest route, the corresponding transition is \( t_j \), then when the reasoning comes to Step \( h \) and Step \( h + 1 \), the token values \( \theta^h_j \) and \( \theta^{h+1}_j \) in the places \( p_j (j = 1, 2, 3, \ldots, n, j \neq i) \) are completely the same, that is, in \( \theta^h \) and \( \theta^{h+1} \), except for the different token values of the sink places, the token values of all the other places are exactly the same, while the equivalent input of each transition is only related to the token value of its input places, while not related to the token values of its output places.

Therefore, in conclusion of the aforementioned, when \( k = h \) and \( k = h + 1 \), the equivalent inputs of \( \rho_h \) and \( \rho_{h+1} \) of all transitions are the same. It can be known that \( \rho_{h+1} = \rho_{h+1} = ((0,1),(0,1),\ldots,(0,1))^T \), according to the Theorem 2, the reasoning ends at this point.

(ii) Then prove that \( k < h + 1 \) is established
When \( k = j, j < h + 1 \), if the equivalent input of each transition that has not been triggered is less than the threshold value of the corresponding transition, then \( \rho_j^* = \rho_k^* \). \( Th = ((0.1),(0.1),…,(0.1))^T \), at this point, the transition shall not be further triggered. As can be known according to equation \((14 \sim 16)\), \( \theta^0 \) no longer changes, therefore, when \( k < h + 1 \), the reasoning may also come to an end.

To sum up the aforementioned, the theorem is proven.

**Theorem 5** The complexity of the reasoning algorithm is \( O(nm^2) \).

Prove Assume there is no loop existing in the MOPN model, then under the normal conditions, the complexity of the reasoning algorithm is \( O(knm) \), in which \( k \) is the number of the loops of the reasoning algorithm, in consideration of the worst scenario, that is, the reasoning is circulated for \( h + 1 \) times ( \( h \) represents the number of transitions of the longest path in the MOPN model), then the general algorithm complexity is \( O((h + 1)nm) = O((m + 1)nm) \), that is, \( O(nm^2) \).

Therefore, assume there are \( n \) propositions and \( m \) rules in the IFPR set \( S \), and there are \( n \) places and \( m \) transitions in the corresponding MOPN model, then the complexity of the reasoning algorithm that is based on MOPN is \( O(nm^2) \).

### 4. Example Validation and Analysis

Rule Set \( S_2 \) is as the following:

\[
R_1; IFd, ANDd_2, ANDd_3, THEN \sim d_6 (\lambda_1 = (0.2,0.6), CF_1 = (0.7,0.2))
\]

\[
R_2; IFd, ANDd_2, THEN d_4 (\lambda_2 = (0.3,0.6), CF_2 = (0.8,0.1))
\]

\[
R_3; IFd, THEN \sim d_2, ANDd_2 (\lambda_3 = (0.1,0.7), CF_3 = (0.6,0.2))
\]

\[
R_4; IFd, AND \sim d_2, THEN d_3 (\lambda_4 = (0.2,0.5), CF_4 = (0.7,0.1))
\]

\[
R_5; IFd, OR \sim d_3, THEN d_4 (\lambda_5 = (0.1,0.8), CF_5 = (0.5,0.3))
\]

\[
R_6; IF \sim (\sim d_6) THEN d_6 (\lambda_6 = (0.5,0.4), CF_6 = (1,0))
\]

In which \( R_\circ \) can be equivalent to the following two rules:

\[
R_1'; IFd, THEN d_{10} (\lambda_1^1 = (0.1,0.8), CF_1^1 = (0.5,0.3))
\]

\[
R_2'; IF \sim d, THEN d_{10} (\lambda_2^1 = (0.1,0.8), CF_2^1 = (0.5,0.3))
\]

\( R_6 \) in the MOPN model as shown in Figure 6 can be ignored. The MOPN model of the Rule set \( S_2 \) is shown in Figure 6.

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**Figure 6.** MOPN Model of Rule Set \( S_2 \)

It can be known that \( n = 10, m = 6, \theta^0 = (\theta_1^0, \theta_2^0, \cdots, \theta_6^0)^T = ((\mu_{11}^0, \gamma_{11}^0), (\mu_{12}^0, \gamma_{12}^0), \cdots, (\mu_{n1}^0, \gamma_{n1}^0))^T \)

\[
\theta^0 = (\theta_1^0, \theta_2^0, \cdots, \theta_6^0)^T
\]

\[
= ((\mu_{11}^0, \gamma_{11}^0), (\mu_{12}^0, \gamma_{12}^0), \cdots, (\mu_{n1}^0, \gamma_{n1}^0))^T
\]

\[
= ((0.6,0.3),(0.7,0.1),(0.5,0.3),(0.7,0.1),(0.8,0.1),(0.1),(0.1),(0.1),(0.1),(0.1))
\]
\[
T_h = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)^T = ((0.2, 0.6), (0.3, 0.6), (0.1, 0.7), (0.2, 0.5), (0.1, 0.8), (0.1, 0.8))^T
\]
\[
CF = \text{diag} \left\{ CF_1, CF_2, CF_3, CF_4, CF_5, CF_6 \right\}^T = \text{diag} \left\{ ((0.7, 0.2), (0.8, 0.1), (0.6, 0.2), (0.7, 0.1), (0.5, 0.3), (0.5, 0.3)) \right\}
\]
\[
\theta_h = ((0.1), (0.1), \ldots, (0.1))^T
\]

The reasoning process is as the following:

1. When the reasoning starts, let \( k = 1 \)
   \[
   \rho_1 = \left( \rho_1^T , \rho_2^T , \ldots, \rho_k^T \right)^T = ((0.5, 0.3), (0.5, 0.3), (0.8, 0.1), (0.1), (0.1), (0.1))^T
   \]
   \[
   \rho_1^* = \rho_{1, \text{Th}} = ((0.5, 0.3), (0.5, 0.3), (0.8, 0.1), (0.1), (0.1), (0.1))^T
   \]
   
2. At this point, \( k = 4 \)
   \[
   \rho_4 = ((0.5, 0.3), (0.5, 0.3), (0.8, 0.1), (0.35, 0.44), (0.496, 0.245), (0.48, 0.28))^T
   \]
   \[
   \rho_4^* = \rho_{4, \text{Th}} = ((0.1), (0.1), (0.1), (0.1), (0.1))^T
   \]
   
It can be found through the examples in this paper that, the main differences of the proposed MOPN model and reasoning algorithm from the existing methods are the following:

1. This paper adopts one place to represent the original proposition and the negative proposition at the same time, and this method has not increased the number of the places, so as to avoid the increase of the complexity of the calculation.

2. Step 3 of the algorithm by adding the step "Determine whether the equivalent input of each transition is greater than the former input", has inhibited the repetitiveness of the transition, so as to avoid the repeated reasoning.

3. Step 6 of the algorithm retains the fact, and avoids the loss of the premise condition in the process of reasoning, which is more compliant with the actual reasoning process.

4. In comparison with the reasoning method that is based on the FPN reasoning, the reasoning method based on MOPN as proposed in this paper has overcome the defects of the single membership grade of the FPN reasoning results, and added the non membership grade in the reasoning results, which is exquisite and comprehensive to represent the reasoning result, and more in line with the reality. For example, the true value of the proposition \( d_{10} \) is \( (0.248, 0.4715) \), which indicates that the membership grade of \( d_{10} \) is 0.248, and the non membership grade is 0.4715.
5. CONCLUSION

This paper combines the matrix theory and Petri nets theory, constructs the MOPN model for the knowledge representation and reasoning, which has solved the representation issue of the negative proposition in the MOPN model, the retention of facts in the MOPN reasoning process and the repetitive triggering of the transition, etc. Example shows that the MOPN model constructed in this paper has overcome the defect of the existing single membership grade in the FPN, due to the increased membership grade, the MOPN has more accurate knowledge representation; the reasoning algorithm based on matrix computation is put forward, in the process of reasoning, the graphic description and parallel computing capacity of Petri nets is made full use of, which enables the reasoning to run automatically and thus has improve the efficiency of the reasoning.

REFERENCES


