Enterprise Performance Evaluation Model Based on DEA Algorithm with Decision Making Unit

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Abstract
In order to evaluate the performance of the enterprise effectively and reasonably, a performance evaluation model is proposed based on optimized DEA algorithm with decision making units. First, a second goal is set on the basis of classic DEA algorithm, and the only public weight is further determined to make the minimizing sum of changes between the efficiency of public weights and the classic DEA algorithm. And then weaken self-interest factor’s influence on the evaluation conclusion in the basic DEA evaluation by using the cross efficiency method. According to the actual situation of objects, give weights for two groups’ efficiency respectively. The final efficiency of units is expressed by the linear combination of the two groups of efficiency and the weights. Simulations show that the error of the proposed improved model is relatively small, the sorting result take into account the organization efficiency maximization, self-evaluation of each unit and group evaluation at the same time, and highlight the evaluation target, which is scientific and effective.

Key words: Performance Evaluation, DEA Algorithm, Decision Making Unit, Cross Efficiency Method, Linear Combination.

1. INTRODUCTION
We have entered into a new economy era with the characteristics of economic globalization, highly informationization and technicalization, and the increasingly fierce market competition and the complexity of the external environment brings new challenges to the enterprise. To obtain long-term development, enterprises must equip themselves with advanced technologies and methods, to prompt management. Business performance evaluation is not only beneficial to understand the internal and external environment of enterprises, an operator clearly know the flow of funds, and promote the optimal allocation of resources, so as to improve the management and the economic efficiency and core competitiveness, to further coordinate the interests relation between the owner and operator, build mutual trust and win-win cooperation mechanism, and is important to establish a reasonable incentive and constraint mechanism for enterprises (Bernadin and Simerly, 2003).

Seen form the domestic and foreign academic circles, performance evaluation is carried on the thorough analysis with the different views and different methods, as well as fusion penetration of disciplines, such as economy and management knowledge, broaden the depth and breadth of the performance evaluation research. Wu Haiming think financial evaluation index is the beginning and end of the performance evaluation of enterprise strategy, he makes the economic added value as a benchmark of evaluation analysis, using non-financial information reveal the enterprise value creation and its changes, establish the financial statement analysis method based on enterprise strategy for performance evaluating of enterprise strategic (Wu, 2009). Zhao Quan and others establish the enterprise performance evaluation system by means of BP neural network method, and provide the new idea of performance evaluation research (Zhao and Zhao and Wang, 2014). On the basis of analyzing the advantages and disadvantages of EVA and BSC, integrating the guiding of the evaluation system of EVA and enterprise long-term strategic of BSC while BSC as the carrier and EVA as oriented, Yang Chengyan builds a new evaluation system (Yan, 2012). Li Jianli think EVA and BSC two methods to reflect two aspects of the performance evaluation: the performance evaluation of the shareholders to agent and the overall performance evaluation of the agent to agent enterprise, the two methods are effectively fusing in together, to overcome scatter of BSC target and simple EVA target, makes the goal of improving enterprise value more clearly, and can find out the key driving factors to affect enterprise performance, in order to better achieve the business goal (Li, 2011).

Jiang Ronghua analyzed the feasibility of fusion of the enterprise strategic management and performance management, and the highest strategic goals are divided into specific departments from the strategic target of quantitative, based on the theory of the balanced scorecard. He sets up key performance indicators of various departments on the four dimensions, and establishes an evaluation index model for execution progress, and the automation of the performance evaluation system is realized (Jiang and Li, 2008). Performance comprehensive evaluation index system which is designed by Worsley (Wang, 2015) is divided into three layers such as target
layer, main factors layer and index layer, main factors layer covers eight indicators: profitability, competitive advantage, debt paying, operation ability, adaptability index of industry, innovation ability, dynamic development capacity and social contribution. Niu Liwen analyzes the practical significance of the enterprise performance evaluation index system, and on this basis, points out six dimensions of evaluation index system, namely while absorbing ideas of balanced scorecard, and with two aspects of other stakeholders and sustainable development, these indicators to evaluate the enterprise from different aspects of the overall business performance, and specific performance evaluation index system is constructed with the actual situation of manufacturing enterprise (Niu and Zhang, 2015). On the basis of absorbing the essence of the factor analysis method, Xu Xuegaos studies operating performance of forty agricultural listed companies, she concludes that the operating performance presents the distribution with “small both sides, big middle” (Xu, 2014). Li Wenhao builds credit rationing model which contains the scale, quality and benefit utility factors, and qualitatively analyzes operating performance of our Banks (Li, 2013).

According to demands of the performance evaluation of enterprise management, we propose a performance evaluation model based on the optimized DEA algorithm with decision making units, and simulations show the effectiveness of the proposed method.

2. VALIDITY ANALYSIS OF DEA ALGORITHM

2.1. Theory of Model

Set there are \( n \) DMUs, \( (1 \leq j \leq n) \), input and output vectors of DMU \( j \) are as follows.

\[
\begin{align*}
    x_j &= (x_{j1}, x_{j2}, \ldots, x_{jm})^T > 0, j = 1, 2, \ldots, n \\
    y_j &= (y_{j1}, y_{j2}, \ldots, y_{mo})^T > 0, j = 1, 2, \ldots, n
\end{align*}
\]  

(1)

Its input and output weights are as follows.

\[
\begin{align*}
    v &= (v_1, v_2, \ldots, v_m)^T \\
    u &= (u_1, u_2, \ldots, u_r)^T
\end{align*}
\]  

(2)

Define that each decision making unit DMU \( j \) has a corresponding efficiency evaluation index as follows.

\[
h_j = \frac{u^T y_j}{v^T x_j} = \frac{\sum_{i=1}^{s} u_i y_{ij}}{\sum_{j=1}^{m} v_i x_{ij}}
\]  

(3)

Take weight coefficient \( v \) and \( u \) appropriately, making \( h_j \leq 1, j = 1, 2, \ldots, n \).

For the efficiency evaluation on the No. \( j_0 \) decision making unit, generally speaking, the bigger \( h_{j_0} \) show that \( DUM_{j_0} \) can use a relatively small input while obtain a relatively more output.

The efficiency index of No. \( j_0 \) decision making unit as the goal, the efficiency indexes of all decision making units as constraint, we can construct the following model.

\[
\begin{align*}
    \max h_{j_0} &= \frac{\sum_{i=1}^{s} u_i y_{j_0}}{\sum_{i=1}^{m} v_i x_{j_0}} \\
    \sum_{i=1}^{s} u_i y_{ij} &\leq 1 \\
    \sum_{i=1}^{m} v_i x_{ij} &\leq 1
\end{align*}
\]  

(4)

The planning model is a fractional programming with Charnes-Cooper, and make \( t = \frac{1}{v^T x_{j_0}}, w = tv, \mu = tu \), we can get the linear programming model \( P \) as the equation (6) by the equation (5).
\[ t = \frac{1}{v'x_0} \Rightarrow w'x_0 = 1 \quad (5) \]

\[
\begin{align*}
\max h_{pr} = \mu' y_0 &= V_r \\
\min w'x_j - \mu' y_j &\geq 0 \\
w'x_0 &= 1 \\
w &\geq 0, \mu &\geq 0
\end{align*}
\quad (P)
\]

Dual planning \( D \) for planning \( P \) is as follows.

\[
\begin{align*}
\max \theta &= V_D \\
\sum_{i=1}^{n} \lambda_i x_j &\leq \theta x_0 \\
\sum_{j=1}^{m} \lambda_j y_j &\geq y_0 \\
\lambda_j &\geq 0
\end{align*}
\quad (D)
\]

The above planning \( D \) is defined as the dual planning of programming \( P \) directly.

### 2.2. Efficiency Analysis of DEA Algorithm

Different production activities need different resources, in order to illustrate the economic effectiveness of DEA model simply, assume that the output of a production activity as \( Y \) only needs two resources \( X_1 \) and \( X_2 \). For the same output, its input indicators also have different ways of investment, the DEA validity is not the same with the different proportion of input.

![Figure 1. Frontier of data envelopment](image-url)

As shown in figure 1, A, B, C, D and E for five different input modes, known as the five different decision making units. A, B, C, D are in a continuous line, and E is out the broken line, and the line is called the production frontier in the field of DEA, is same production combination of a series of line segments, that is to say, decision making units A, B, C, D in the production frontier, E is envelope within the production frontier. Economically, decision making units A, B, C, D are effective for technology, and E is invalid. Invalid technology so-called refers to the output is not achieve the optimal in a given inputs, inputs too much and too little outputs, only reduces one or more inputs or increase outputs to make it into an effective technology. Find changing the technology benefit of E from the figure 1: connect E point and the origin point, crossing production frontier on D point, and D point is a effective decision making unit point corresponding to the decision making unit E. Improve E along the direction of the D gradually by adjusted, to achieve effective technology. Compared with the E, same outputs point D has fewer inputs, so the resources have get reasonable use.
3. PERFORMANCE EVALUATION MODEL BASED ON DEA ALGORITHM WITH OPTIMIZATION OF DECISION MAKING UNITS

3.1. Refactoring Goal of Decision Units

Because the efficiency of the classical DEA method is get form the most efficient of each decision making unit, and constraint condition is all efficiency values are less than 1. So, as the increase of the constraint conditions of any other, make the efficiency based on public weights is less than or equal to the efficiency by classical DEA efficiency method. And each decision unit is always hoping to own maximize efficiency, the decision making units don’t expect that gap is too big. So we set a second goal, further determine the only public weight, make the sum of the minimizing change between efficiency based on public weights and the classic DEA method.

First of all, we must find out the efficiency value of the classic DEA model. According to the demand of the performance evaluation of enterprise management, establishment of the classic DEA model are as follows.

\[
\begin{align*}
\max \theta^*_c &= \frac{\sum_{i=1}^{n} u_i y_i}{\sum_{i=1}^{m} v_i x_i} \\
\theta^*_j &= \sum_{i=1}^{n} u_i y_{ij} \\
\theta^*_{n+1,j} &= \sum_{i=1}^{n} v_i x_{ij}
\end{align*}
\]

(8)

In the equation (8), get the efficiency value \( \theta^*_j \) of No. \( j \) decision unit \( DMU_j \), efficiency value \( \theta^*_{n+1,j} \) of the unit \( j \) is obtained by equation (8) based on the maximum total efficiency, the second decision target is to make the sum of the change is the minimum between \( \theta^*_j \) and \( \theta^*_{n+1,j} \).

After getting the relative efficiency \( \theta^*_{n+1,j} \) of unit \( j \) based on public weights and the relative efficiency \( \theta^*_j \) of unit \( j \) based on classic DEA, set up DEA model which makes minimizing changes of the sum of both to determine the only one common set of public weights, specific in equation(9).

\[
\begin{align*}
\min \sum_{j=1}^{n} (\theta^*_j - \theta^*_{n+1,j}) \\
\theta^*_{n+1,j} &= \sum_{i=1}^{n} u_i y_{ij} \\
\theta^*_{n+1,n+1} &= \theta^*_{n+1}
\end{align*}
\]

(9)

In the equation (9), \( \theta^*_{n+1} \) is relative efficiency of unit \( n+1 \), which is the relative efficiency of combined virtual unit. Through the transformation of equation (9), we get equations (10) and (11) and (12).

\[
\begin{align*}
\min \sum_{j=1}^{n} (\theta^*_j) - \sum_{i=1}^{n} u_i y_{ij} \\
\sum_{i=1}^{n} u_i y_{n+1} - \theta^*_{n+1} \sum_{i=1}^{n} v_i x_{n+1} = 0 \\
\sum_{i=1}^{n} u_i y_{ij} - \sum_{i=1}^{n} v_i x_{ij} \leq 0
\end{align*}
\]

(10)

(11)

(12)

By solving equations (10) ~ (12), we can get a set of public weights \( u_i \) and \( v_i \), which can remove
different preference of different decision making units for weights and accepted by all units to a great extent. We define the relative efficiency values of decision making units as the ratio of each output weighted summation with each input weighted summation, than calculated \( n \) times, the efficiency value of each decision-making unit \( DMU_j \) based on the only public weight is as follows.

\[
E_j = \frac{\sum_{i=1}^{n} u_i y_{ij}}{\sum_{i=1}^{n} v_i x_{ij}}
\]  

\( \text{(13)} \)

3.2. Weaken Self-interest Factors Based on Cross Efficiency

Total efficiency and self-evaluation efficiency are considering the whole organization and each individual, but have no regard for the evaluation between individuals. Peer evaluation is one very important idea in enterprise performance evaluation. In order to solve a single subject in the basic DEA evaluation is unfairness, we weaken self-interest factors’ influence on the evaluation conclusion in the basic DEA evaluation by the cross efficiency method, to make the evaluation conclusions more close to the actual situation.

Calculate the cross efficiency values of each decision making unit by the weights which obtained by the classic DEA model, and the cross efficiency of decision making units \( DMU_j \) that evaluated by the defining decision making units \( DMU_k \) is as follows.

\[
\theta_{jk} = \frac{\sum_{i=1}^{n} u_{ik} y_{ij}}{\sum_{i=1}^{n} v_{ik} x_{ij}}
\]  

\( \text{(14)} \)

All decision making units are getting cross efficiency by \( n \) evaluation decision units through evaluating of DEA model, and build cross efficiency matrix, and then the average cross efficiency value is calculated. The average cross efficiency value \( A_j \) of decision making units is defined as follows.

\[
A_j = \frac{\sum_{k=1}^{n} \theta_{jk}}{n}
\]  

\( \text{(15)} \)

Cross efficiency method allows all the evaluation units participate in efficiency evaluation, is a group evaluation essentially, its advantage is gathering more information, weakening self-interest tendency in the basic DEA evaluation makes the evaluation conclusion more likely to be accepted.

3.3. Evaluation Sort of Decision Making Units

For more balance of the efficiency evaluation of the organization and more considerate, we also find out the cross efficiency based on group evaluation (peer evaluation). Second focus of two groups efficiency is different, according to the actual situation of enterprises, we can give weights for two group efficiency, and according to the empowerment is heavy, the final efficiency of unit by the linear combination of the two groups of efficiency and the weight.

According to the different situation and evaluation purpose of each enterprise, the weights of the efficiency based on the public weight and the average cross efficiency value of decision making units \( DMU_j \) are given respectively as \( \alpha \) and \( \beta \), the final efficiency value of \( DMU_j \) is as follows.

\[
\begin{align*}
E_j^* &= \alpha E_j + \beta A_j \\
\alpha + \beta &= 1
\end{align*}
\]  

\( \text{(16)} \)

Finally, the relative efficiency \( E_j^* \) of each decision making unit is obtained by equation (16), according to the size of \( E_j^* \) to evaluate and sort the decision-making unit.

From what has been discussed above, sorting of decision making units can be divided into the following five steps:

1. Establish a new evaluation system. Make organization as a sum of decision making unit \( DMU_{n+1} \), which is added to the evaluation system. Thus construct a new evaluation system containing \( n+1 \) decision-making units.
2. According to the new evaluation system to establish a new DEA model and solving, the objective
function is to maximize total efficiency.

(3) Build the second decision objective, namely get a minimum sum of the efficiency of the weights based on public income and the classic DEA method. Calculate the weight $u_r (r = 1, ..., s)$ and $v_i (i = 1, ..., m)$.

(4) Use only certain public weight $u_r (r = 1, ..., s)$ and $v_i (i = 1, ..., m)$ to calculate the maximum efficiency value $E_j$ of each decision-making unit based on total virtual units.

(5) Establishing cross efficiency model and solving, we get the average cross efficiency value $A_j (j = 1, ..., n)$.

(6) The weights of $E_j$ and $A_j$ are given as $\alpha$ and $\beta$ respectively, and the efficiency values $E'_j$ are obtained according to the equation (16), finally sort all decision making units by the size of $E'_j$.

4. INSTANCE ANALYSIS

To verify the performance of the improved algorithm proposed in this paper, we get simulation test. An enterprise actual data shows that the proposed new sorting method is scientific. The instance contains 10 decision making units, each decision unit has two inputs and two outputs, as shown in table 1.

Table 1. Instance data

<table>
<thead>
<tr>
<th>Decision Making Units</th>
<th>Input1</th>
<th>Input2</th>
<th>Output1</th>
<th>Output2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DMU_1$</td>
<td>9</td>
<td>9</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$DMU_2$</td>
<td>12</td>
<td>8</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$DMU_3$</td>
<td>7</td>
<td>12</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$DMU_4$</td>
<td>6</td>
<td>10</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$DMU_5$</td>
<td>10</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$DMU_6$</td>
<td>8</td>
<td>10</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$DMU_7$</td>
<td>12</td>
<td>10</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$DMU_8$</td>
<td>14</td>
<td>6</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>$DMU_9$</td>
<td>12</td>
<td>12</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>$DMU_{10}$</td>
<td>8</td>
<td>8</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Sum</td>
<td>98</td>
<td>90</td>
<td>37</td>
<td>33</td>
</tr>
</tbody>
</table>

In the table 1, $DMU_1$~$DMU_{10}$ are ten decision making units of original evaluation system. Input1 and Input2 as two inputs for each decision making unit, Output1 and Output2 as two outputs for each decision making unit, the Sum as input and output of total virtual units, namely the Sum of the input and the output for the other 10 decision-making units.

Table 2. Efficiency based on public weights

<table>
<thead>
<tr>
<th>Decision Making Units</th>
<th>$E'_j$</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DMU_1$</td>
<td>0.33001</td>
<td>10</td>
</tr>
<tr>
<td>$DMU_2$</td>
<td>0.40058</td>
<td>9</td>
</tr>
<tr>
<td>$DMU_3$</td>
<td>0.40064</td>
<td>8</td>
</tr>
<tr>
<td>$DMU_4$</td>
<td>0.99999</td>
<td>3</td>
</tr>
<tr>
<td>$DMU_5$</td>
<td>0.95384</td>
<td>4</td>
</tr>
<tr>
<td>$DMU_6$</td>
<td>0.62416</td>
<td>6</td>
</tr>
<tr>
<td>$DMU_7$</td>
<td>1.00000</td>
<td>1</td>
</tr>
<tr>
<td>$DMU_8$</td>
<td>1.00000</td>
<td>1</td>
</tr>
<tr>
<td>$DMU_9$</td>
<td>0.45895</td>
<td>7</td>
</tr>
</tbody>
</table>
According to table 1, calculate maximum efficiency $E_j$ of each decision-making unit based on the efficiency of combined virtual unit, which is shown in table 2.

Calculate the relative efficiency $E'_j$, the results are shown in table 3.

<table>
<thead>
<tr>
<th>Decision Making Units</th>
<th>$E'_j$</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DMU_1$</td>
<td>0.29164</td>
<td>10</td>
</tr>
<tr>
<td>$DMU_2$</td>
<td>0.34175</td>
<td>9</td>
</tr>
<tr>
<td>$DMU_3$</td>
<td>0.37382</td>
<td>8</td>
</tr>
<tr>
<td>$DMU_4$</td>
<td>0.90523</td>
<td>2</td>
</tr>
<tr>
<td>$DMU_5$</td>
<td>0.87166</td>
<td>3</td>
</tr>
<tr>
<td>$DMU_6$</td>
<td>0.57901</td>
<td>6</td>
</tr>
<tr>
<td>$DMU_7$</td>
<td>0.92144</td>
<td>1</td>
</tr>
<tr>
<td>$DMU_8$</td>
<td>0.84392</td>
<td>4</td>
</tr>
<tr>
<td>$DMU_9$</td>
<td>0.45041</td>
<td>7</td>
</tr>
<tr>
<td>$DMU_{10}$</td>
<td>0.83259</td>
<td>5</td>
</tr>
</tbody>
</table>

The above results are compared with the standard DEA model, and the error is shown in Figure 2.

![Figure 2. Comparison of evaluation error](image)

Seen from the above results, the proposed improved model error is relatively small, the sorting result take into account the organization efficiency maximization, each unit of the self evaluation and group evaluation, and highlight the evaluation target, so it is scientific and effective.

5. CONCLUSIONS

As a variety of complex external environment, it is imperative that enterprises to improve their own comprehensive and business performance are an important level of the enterprise ability. Therefore, build scientific and reasonable performance evaluation system for enterprises, for comprehensive evaluation of the operation of the implementation, is not only improve the development of enterprises, is also meet objective requirement of market competition. According to the demand of the performance evaluation of enterprise management, a performance evaluation model is proposed based on optimized DEA algorithm by decision making units, simulations show that compared with the standard model, the improved model has higher precision of evaluation.
REFERENCES


