**Numerical Calculation of Induction Logging Response in Cased Hole**

Yinchuan Wu*
Key Laboratory of Photoelectric Logging and Detecting of Oil and Gas of Ministry of Education, Xi'an Shiyou University, Xi'an 710065, Shaanxi, China
*Corresponding author (E-mail: wuyinchuan@163.com)

Jiatian Zhang
Key Laboratory of Photoelectric Logging and Detecting of Oil and Gas of Ministry of Education, Xi'an Shiyou University, Xi'an 710065, Shaanxi, China

**Abstract**

Induction logging is an effective method of measuring formation conductivity in open hole. However, when the metallic pipes are inserted into the borehole, the standard induction logging is found to be invalid. In this paper, a system of single-well through-casing induction logging is modeled, and the formulas of the electromagnetic and the induction voltage are derived in cased hole. Moreover, in order to identify the formation conductivity surrounding the casing, a phase difference measuring method is proposed. Calculation results show that: the formation surrounding the casing can be identified by the phase difference. The phase difference will vary linearly with the conductivity of the formation. The lower the formation conductivity is, the smaller the phase difference is, and the harder the detection is. The appropriate receiver location and the appropriate excitation frequency can be obtained by the curves of the phase difference and voltage amplitude.

**Key words:** Induction Logging, Electromagnetic Field, Formation Conductivity, Cased hole

**1. INTRODUCTION**

The value of open hole induction logs as the primary source for determining hydrocarbon saturation is well established for reservoir formation evaluation in the oil industry (Wu, Gong and Pang, 2003). Casing is commonly used to avoid collapse of well in oil fields by inserting metallic pipes into the borehole. Most oil wells are cased with pipes except for exploratory and newly drilled wells (Wu, Guo and Zhang, 2014). Steel is the most commonly used material for the pipes. Since steel casing is more electrically conductive (106 S/m is a typical conductivity) and magnetically conductive (the range of relative permeability is 40-110) than the formation around the borehole, electromagnetic (EM) signals from the surrounding formation undergo sizable attenuation as they are transmitted across the casing (Yang, 2000; Lu, Liu and Chen, 2012; Li, 2015). Standard EM logging devices operate at frequencies (10 kHz-100 kHz) too high for their signals to penetrate casings. Once a borehole is cased, the formation behind the casing is virtually inaccessible to standard induction logging methods. Fortunately, some studies indicate that EM signals through steel casing can be detected at low frequencies (Wei, Zhang and Ling, 1999; Hu, Xu and Wang, 2007).

Induction surveys all share the same physical principles (Kaufman and Dashhevsky, 2013). A transmitter, usually a multi-turn coil of wire, carries an alternating current of frequency. This creates a time varying magnetic field in the surrounding formation which in turn, by Faraday’s law, induces an electromotive force (EMF). This EMF drives currents in the formation which are basically proportional to the formation conductivity (Li, 2016). Finally a receiver is positioned in the same hole as the transmitter. The receiver measures the magnetic field arising from the transmitter and the secondary or induced currents in the formation. In cased hole, however, not only the formation but also the casing causes the magnetic fields.

The theoretical studies of the single-well through-casing induction measurement proposed the mathematical formulation (using a large-loop transmitter located on the surface and a receiver, or receiver array, lowered into the well), the computational model, excitation frequency range, the effect of non-uniform casing properties (Augustin, Kennedy, Morrison and Lee, 1989; Kong, Roth, Olsen and Stalheim, 2009; Lee, Kim and Uchida, 2005; Kim and Lee, 2006). Since any variations in the rock conductivity can be masked by even minute changes in the casing dimensions and material properties (conductivity and permeability), a spatial low-pass filtering and measurement of the casing properties have been proposed for the casing effect correction (Kim and Lee, 2006; Vasic and Bilas, 2012; Gao, 2014). The experimental verification of the through-casing induction measurement on a scaled laboratory mode of a borehole lined with nonmagnetic metal casing surrounded with the low-conductive medium is presented, and the measurement results are in agreement with the theoretical predictions (Vasic, Bilas and Peyton, 2013).
In this paper, a system of single-well through-casing induction logging is modeled, and the mathematical formulations (the receiver is positioned the same hole as the transmitter) of electromagnetic fields in a cased borehole surrounded by uniform whole space are developed. Moreover, the distributions of the electromagnetic field and the induced voltage are studied through model calculation, and the phase difference of induced voltage and the apparent conductivity are defined. Furthermore, according to the results, the relationship between the phase difference and the formation conductivity can be grasped, and the appropriate frequency and the distance between the transmitter coil and the receiver coil can be found.

2. ELECTROMAGNETIC IN CASED HOLE

2.1. Induction Logging Model

The model of the problem is given in Figure 1. Because of the axial symmetry of the problem domain, it is convenient to employ the cylindrical coordinate system \((r, \phi, z)\). A transmitter coil \(T\) with radius \(a\) is positioned at \(z = 0\), and a receiver coil \(R\) with same radius \(a\) is positioned at height \(L\). The casing (medium II) has inner radius \(b\), outer radius \(d\) and wall thickness \(b-d\). The tube material is assumed to be homogenous with electrical conductivity \(\sigma_2\) and magnetic permeability \(\mu_2\) (\(\mu_2 = \mu_{2r}\mu_0\), \(\mu_{2r}\) and \(\mu_0\) are relative magnetic permeability and vacuum magnetic permeability respectively). The homogenous formation (medium III) surrounding the casing has electrical conductivity \(\sigma_3\) and magnetic permeability \(\mu_3\) (\(\mu_3 = \mu_{3r}\mu_0\)). And \(\sigma_1\) and \(\mu_1\) (\(\mu_1 = \mu_{1r}\mu_0\)) are electrical conductivity and magnetic permeability of the borehole (medium I), respectively.

![Figure 1. Model of induction logging in cased hole](image)

2.2. Electromagnetic in the Medium

When the displacement current density is ignored, Maxwell’s equations (Hayt and Buck, 2009) are generally expressed in frequency domain as

\[
\nabla \times \mathbf{H} = \sigma \mathbf{E} + \mathbf{J}_T \\
\nabla \times \mathbf{E} = -j\omega \mu \mathbf{H}
\]

where \(\mathbf{E}\) is the electric field, \(\mathbf{H}\) the magnetic field, \(\sigma\) the conductivity, \(\mu\) the magnetic permeability, \(\omega\) the current angular frequency, \(\mathbf{J}_T\) the source current distribution. Because of axial symmetry of the problem, the fields have specific directional components as

\[
\mathbf{J}_T = J_T(0, J_{T\phi}, 0) \\
\mathbf{E} = E(0, E_{\phi}, 0)
\]

and

\[
\mathbf{H} = H(H_r, 0, H_z)
\]

Application of the curl operator in cylindrical coordinates in equations (1) and (2) gives

\[
\frac{\partial E_\phi}{\partial x} = j\omega \mu H_r \\
\frac{1}{r}\frac{\partial (rE_\phi)}{\partial r} = -j\omega \mu H_z
\]

and

\[
\frac{\partial H_r}{\partial x} - \frac{\partial H_\phi}{\partial r} = \sigma E_\phi + J_{T\phi}
\]

After elimination of \(H_z\) and \(H_r\) from equations (6), (7) and (8), \(E_\phi\) satisfies
\[
\frac{\partial^2 E_y}{\partial z^2} + \frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_r}{\partial r} + (k^2 - \frac{1}{r^2})E_\phi = j \omega \mu I_\phi
\]

where \( k = \sqrt{-j \omega \mu \sigma} \), \( k \) is called the propagation constant in the medium. The homogeneous form of equation (9) can be solved by separation variables. The solution has the form

\[
E_\phi (r, z) = -\frac{j \omega M}{2 \pi r} \int_0^\infty \left[ C_1 (\lambda r) + DK_1 (\lambda r) \right] \cos (\xi z) d\xi
\]

where \( \lambda^2 = \xi^2 - k^2 \) and \( M = \pi a^2 N_\tau l_\tau \), \( M \) is the magnetic dipole moment of the transmitter coil, \( N_\tau \) the number of turns of the transmitter coil, \( I_\tau = \int_{r_\tau} r_\tau ds \) the current of the transmitter coil, \( C \) and \( D \) undetermined coefficients, \( I_1 (\lambda r) \) and \( K_1 (\lambda r) \) the modified Bessel functions of the first and second kinds of order 1. Using the relation between \( H_r \) and \( E_\phi \) in equation (6), \( H_r (r, z) \) is expressed by

\[
H_r (r, z) = \frac{M}{2 \pi r} \int_0^\infty \xi [C_1 (\lambda r) + DK_1 (\lambda r)] \sin (\xi z) d\xi
\]

Using the relation between \( H_z \) and \( E_\phi \) in equation (7), \( H_z (r, z) \) is expressed by

\[
H_z (r, z) = \frac{M}{2 \pi r} \int_0^\infty \lambda [C_1 (\lambda r) - DK_0 (\lambda r)] \cos (\xi z) d\xi
\]

In medium I (borehole), since the condition (1) is satisfied, the coefficient \( D \) in equations (10), (11) and (12) equals zero. The electromagnetic fields in medium I (borehole) are expressed by

\[
E_\phi (r, z) = -\frac{j \omega M}{2 \pi r} \int_0^\infty [C_1 (\lambda r) + \lambda K_1 (\lambda r)] \cos (\xi z) d\xi
\]

\[
H_r (r, z) = \frac{M}{2 \pi r} \int_0^\infty \xi [C_1 (\lambda r) + \lambda K_1 (\lambda r)] \sin (\xi z) d\xi
\]

\[
H_z (r, z) = \frac{M}{2 \pi r} \int_0^\infty \lambda [C_1 (\lambda r) - \lambda K_0 (\lambda r)] \cos (\xi z) d\xi
\]

In medium II (casing), the equations are

\[
E_\phi (r, z) = -\frac{j \omega M}{2 \pi r} \int_0^\infty [C_2 (\lambda r) + D_2 K_1 (\lambda r)] \cos (\xi z) d\xi
\]

\[
H_r (r, z) = \frac{M}{2 \pi r} \int_0^\infty \xi [C_2 (\lambda r) + D_2 K_1 (\lambda r)] \sin (\xi z) d\xi
\]

\[
H_z (r, z) = \frac{M}{2 \pi r} \int_0^\infty \lambda [C_2 (\lambda r) - D_2 K_0 (\lambda r)] \cos (\xi z) d\xi
\]

In medium III (formation), since the condition (2) is satisfied, the coefficient \( C \) in equations (15), (16) and (17) equals zero. The electromagnetic fields in medium III (formation) satisfy

\[
E_\phi (r, z) = -\frac{j \omega M}{2 \pi r} \int_0^\infty D_3 K_1 (\lambda r) \cos (\xi z) d\xi
\]

\[
H_r (r, z) = \frac{M}{2 \pi r} \int_0^\infty \xi D_3 K_1 (\lambda r) \sin (\xi z) d\xi
\]

\[
H_z (r, z) = -\frac{M}{2 \pi r} \int_0^\infty \lambda D_3 K_0 (\lambda r) \cos (\xi z) d\xi
\]

At the two interfaces, the condition (3) is satisfied. Applying the boundary conditions yield the system equations, which can be obtained using matrix algebra

\[
\begin{bmatrix}
\mu_1 I_1 (\lambda_1) & -\mu_2 I_1 (\lambda_2) & -\mu_3 K_1 (\lambda_3)
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
D_1 \\
D_2
\end{bmatrix} = \begin{bmatrix}
-\mu_1 K_1 (\lambda_1)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\lambda_1 I_0 (\lambda_1) & -\lambda_2 I_0 (\lambda_2) & -\lambda_3 K_0 (\lambda_3)
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
D_1 \\
D_2
\end{bmatrix} = \begin{bmatrix}
-\lambda_1 K_0 (\lambda_1)
\end{bmatrix}
\]

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Solving the matrix equation (24)

$$G_1 = \frac{A_1(A_1B_1 + A_2B_2)}{A_1B_3 - A_2B_3}$$

where

$$A_1 = \mu_2\lambda_3K_0(\lambda_3d)K_1(\lambda_2d) - \mu_3\lambda_2K_0(\lambda_3d)K_1(\lambda_3d)$$
$$A_2 = \mu_2\lambda_3l_2(\lambda_2d)K_0(\lambda_3d) + \mu_3\lambda_2l_0(\lambda_2d)K_1(\lambda_3d)$$
$$A_3 = \mu_2\lambda_3l_1(\lambda_2b)K_0(\lambda_3b) + \mu_3\lambda_2l_0(\lambda_2b)K_1(\lambda_2b)$$

Therefore, the expressions for the electromagnetic fields in medium I are obtained. And the electromotive force of the receiver can be expressed by

$$E(a, L) = \oint E \cdot dl = \oint E_{1\phi}(a, L) dl = 2\pi a N_R E_{1\phi}(a, L)$$

where $N_R$ is the number of turns of the receiver coil.

3. RESPONSE CALCULATION AND RESULTS ANALYSIS

3.1. Axial Magnetic Field in Cased Hole

In order to investigate the axial magnetic field in cased hole, we chose the model shown in figure 1. The medium I (borehole) has an electrical conductivity $\sigma_1 = 0$ S/m and a magnetic permeability $\mu_1 = \mu_0 = 4\pi \times 10^{-7}$ H/m. The inner casing radius $b$ is 0.1m and the outer casing radius $d$ is 0.11m. The medium II (casing) has an electrical conductivity $\sigma_2 = 1 \times 10^6$ S/m and a magnetic permeability $\mu_1 = 100\mu_0$. The medium III (formation) has an electrical conductivity $\sigma_3 = 0.1 \sim 5$ S/m and a magnetic permeability $\mu_1 = \mu_0$. The source used is a loop of wire of radius $a = 0.03$m and the number of turns $N_R = 100$, carrying 1A current at 100Hz. Vertical magnetic fields $H_{1z}(0, z)$ are measured in the cased well.

Figure 2 and figure 3 show real parts and imaginary parts of $H_{1z}(0, z)$ fields. The formation conductivity is set at 0.1 S/m, 0.5S/m, 1S/m and 5S/m. The real part of magnetic field will be on the same order of magnitude as the imaginary part of magnetic field at the same z location, whereas the real part of axial magnetic field is $10^3 \sim 10^5$ times greater than the imaginary in borehole. In spite of different formation surrounding the casing, in figure 2, the imaginary part curves of axial magnetic fields overlap. In figure 3, the real part curves of axial magnetic fields reflect formation small changes when $z > 3$ m and $\sigma_3 > 0.5$ S/m. So, it is very difficult to detect the formation conductivity in steel casing hole by standard induction logging technology, whereas it is easy to distinguish the formation through measuring the imaginary part of the magnetic field in borehole(Zhang, Liu, and Wu, 2002).

Figure 2. Real parts of axial magnetic fields for four different electrical conductivity of the formation (0.1 S/m, 0.5S/m, 1S/m and 5S/m).
Figure 3. Imaginary parts of axial magnetic fields for four different electrical conductivity of the formation (0.1 S/m, 0.5S/m, 1S/m and 5S/m).

3.2. Induced Voltage of the Receiver Coil

In figure 1, the medium I (borehole) has an electrical conductivity $\sigma_1 = 0$ S/m and a magnetic permeability $\mu_1 = \mu_0$, the medium II (casing) has an electrical conductivity $\sigma_2 = 2 \times 10^6$ S/m and a magnetic permeability $\mu_1 = 100\mu_0$, the medium III (formation) has an electrical conductivity $\sigma_3 = 0.05$ S/m and a magnetic permeability $\mu_1 = \mu_0$, and the coil has an radius $a = 0.05$ m, the number $N_T = N_R = 10000$, the transmitter current 1 A and the frequency 100 Hz.

Figure 4 and figure 5 show modulus and phase of inductive voltage in cased hole. Figure 4 shows that the modulus curves of inductive voltage coincide under different formation conditions, in other words, it is very difficult to identify the various formations surrounding the casing by measuring the voltage amplitude. Fortunately, in figure 5, when $z > 2.5$ m, the phase of the induced voltage vary with the formation conductivity, the phase is smaller for higher formation conductivity and the changes are more clear for longer distance of the receiver. So, we can identify the formation surrounding the steel casing by detecting the phase of the induced voltage.

Figure 4. Modulus of inductive voltage for five different electrical conductivity of the formation (0S/m, 0.1 S/m, 0.5 S/m, 1S/m and 5S/m)

Suppose the phase $\varphi_C$ of the casing alone and the phase $\varphi_F$ of the formation alone could be combined to yield the phase $\varphi_{CF}$ of the casing-in-formation. So the total phase $\varphi_{CF}$ can be decomposed into

$$\varphi_{CF} = \varphi_C + \varphi_F$$

The phase $\varphi_C$ can be calculated when the casing parameters ($b, d, \mu_2, \sigma_2$) can be obtained by other logging methods (Epov, Morozova, Antonov and Kuzin, 2003; Dyakin, Sandovskii and Dudarev, 2004; Chen, Chai, Jin, Li, Feng and Ma, 2005; Wang and Fei, 2007), the phase difference $\Delta\varphi$ is expressed by

$$\Delta\varphi = \varphi_C(\sigma_2; \sigma_3 = 0) - \varphi_{CF}(\sigma_2; \sigma_3)$$

where $\varphi_C(\sigma_2; \sigma_3 = 0)$ is the phase of the casing alone, $\varphi_{CF}(\sigma_2; \sigma_3)$ is the phase of the casing-in-formation.
Substituting the results in figure 5 into equation (29), the phase difference for four different formation is displayed in figure 6. The phase difference is larger for higher formation conductivity and larger coil separations. When the distance $z$ is 5 m, the relationship between the formation conductivity and the phase difference is shown in figure 7. Obviously, the phase difference will vary linearly with the conductivity of the formation, the relationship can be written as

$$\Delta \varphi = 0.477\sigma_f + 0.032$$

(30)

So the apparent conductivity $\sigma_a$ of the formation can be obtained

$$\sigma_a = \frac{\Delta \varphi - 0.032}{0.477}$$

(31)

Figure 5. Phase of inductive voltage for five different electrical conductivity of the formation (0 S/m, 0.1 S/m, 0.5 S/m, 1 S/m and 5 S/m)

Figure 6. Phase difference of inductive voltage for four different electrical conductivity of the formation (0.1 S/m, 0.5 S/m, 1 S/m and 5 S/m)

Figure 7. Conductivity of the formation against the phase difference of voltage

3.3. Choice of Excitation Frequency and Distance between the Coils
It is very important to answer the question of the choice of the excitation frequency and transmitter-receiver distance required for the measurement of formation conductivity. In this section, the medium III has the formation conductivity $\sigma_3 = 1$ S/m, the excitation frequency is 25Hz~400Hz, and other parameters are maintained. The phase difference and induced voltage are depicted in figure 8 and figure 9, respectively, as functions of excitation frequency and distance from the transmitter coil. The phase difference is larger for higher excitation frequencies and larger coil separations in figure 8, whereas the induced voltage rapidly decreases with the frequency and distance in figure 9. From figure 8 and figure 9, the parameters of the frequency and the distance can be determined. If the amplitude of induced voltage is larger than 100$\mu$V and the phase difference is larger than 0.1°, the results can be shown in Figure 10. And the appropriate frequency and distance can be obtained in the area below the 100$\mu$V voltage curve and above the 0.1° phase difference, such as 100Hz and 4m, 50Hz and 5m.

![Figure 8. Phase difference as a function of distance z and excitation frequency](image1)

![Figure 9. Modulus of inductive voltage as a function of distance z and excitation frequency](image2)

![Figure 10. Selection of the excitation frequency and receiver coil location z](image3)

4. CONCLUSIONS

In a single cased hole, the formation conductivity behind the casing is measured by induction logging method. The formation surrounding the steel casing is identified by detecting the phase of the induced voltage. The phase difference will vary linearly with the conductivity of the formation. The lower the formation conductivity is, the smaller the phase difference is, and the harder the formation detection is. According the
curves of phase and induced voltage amplitude, the excitation frequency and the coils distance can be obtained in the area.

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