Pricing Binary Options Based on Fuzzy Number Theory

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Abstract
Options pricing model parameters are inherently imprecise due to fluctuations in the real-world financial market. Traditional option pricing methods do not account for the uncertainty in parameters, but the fuzzy set theory may be applicable. This paper proposes a cash-or-nothing European call binary option pricing model based on the hypothesis that the underlying asset price, risk-free rate of interest, and volatility all are uncertain. We present the fuzzy pricing model of the cash-or-nothing call binary option under the fuzzy environment. Two numerical examples presented in the paper illustrate the rationality and effectiveness of the fuzzy option pricing model.

Keywords: Binary Option, Cash-Or-Nothing Option, Fuzzy Number, Fuzzy Option Pricing

1. INTRODUCTION
An option is a contract which gives its holder the right (but not the obligation) to buy or sell an underlying asset at a predetermined price on a specified date. Options can be categorized as “call options” and “put options”. The call option gives the holder the right to buy an underlying asset at a predetermined price; the put option gives the holder the right to sell an underlying asset at a predetermined price. Options first appeared in the early 1970’s as a finance innovation and since then have developed quickly into an efficient approach toward risk hedging.

In effort to satisfy the increasingly diverse needs of investors, “exotic options” with a wide array of characteristics have been designed based on the foundation of the standard contract. Exotic options, put simply, are options that do not share the same characteristics as standard options (Wei et al., 2015). The path-dependent option is a type of exotic option that features payoffs related to the underlying asset price at maturity, or to the price path between the current time and expiration date. The binary option is a path-dependent option that has discontinuous payoffs. The binary option can be used to hedge and speculate; it is very popular in over-the-counter market dealings. There are two main types of binary options: The “cash-or-nothing” binary option, and the “asset-or-nothing” binary option.

The traditional binary option pricing model only applies to scenarios in which the underlying stock price, risk-free interest rate, and price volatility are certain. In the actual financial market, precise option model parameters are not necessarily attainable – it is especially challenging (even impossible) to precisely determine the underlying asset price, risk-free rate of interest, or volatility. Further, investors tend to focus on the range within which option prices tend to oscillate, while generally neglecting the precise option price. In short: The traditional option pricing model does not accurately reflect the real-world financial market.

The fuzzy sets theory first proposed by Zadeh can be utilized to solve problems that feature uncertainty (Zadeh, 1965). Ribeiro et al., in a notable example of this, addressed a finance engineering problem by applying the fuzzy set theory (Ribeiro et al., 1999). Based on Black-Scholes pricing formula, Wu established a fuzzy pricing model for European options (Wu, 2004; Wu, 2005; Wu, 2007). Considering the randomness and fuzziness under uncertain environments, Yoshida derived a novel method for valuating European option prices by using fuzzy numbers (Yoshida, 2003). Lee et al. developed a fuzzy binomial European option pricing approach in the CRR model and conducted an empirical analysis by taking S&P 500 index options as an example (Lee et al., 2005). Thiagarajah et al. built a European options pricing model by using adaptive fuzzy numbers (Thiagarajah et al., 2007), and Xu et al. built a fuzzy pricing model for European options in normal jump-diffusion with uncertain randomness and fuzziness. The pricing model can be considered an extension of the Merton’s option price model (Xu et al., 2009). Yen derived a new option pricing model by introducing a non-uniform self-selective coder (Yen, 2010). Based on the assumption that the underlying asset price, discount rate, the volatility and risk-free rate of interest are all fuzzy numbers, Zhang et al. obtained a fuzzy pricing formula and corresponding algorithm for American options (Zhang et al., 2011).

There have indeed been many valuable contributions to the literature, but to date, there have been relatively few studies on exotic options pricing within fuzzy environments apart from those by Wang et al. (Wang et al., 2014) and Zhan (Zhang, 2014). Recently, Thavaneswaran et al. used fuzzy set theory to price asset-or-nothing
European binary options (Thavaneswaran et al., 2013), but only by fuzzifying the maturity value of the underlying asset price while considering risk-free rate of interest and volatility to be certain, real numbers. In the actual financial market, data such as underlying asset price, risk-free interest rate, and volatility simply cannot be precisely obtained. Our primary goal in conducting the present study was to fill these gaps in knowledge – that is, to build a cash-or-nothing European binary option pricing model by fuzzifying the underlying assets price, risk-free rate of interest, and volatility.

A number of other studies served as inspiration for the present study. Carlsson and Fuller used fuzzy numbers to explore the real option pricing problem and identified the possibilistic mean value and possibilistic variance of fuzzy numbers simultaneously (Carlsson and Fuller, 2001). Thavaneswaran et al. proved the superiority of fuzzy forecasts compared to other techniques per the option pricing problem in the GARCH model by applying the weighted possibilistic moments of fuzzy numbers (Thavaneswaran et al., 2009). Zmeskal proposed a fuzzy binomial model for pricing American real options (Zmeskal, 2010). Guerra et al. and Chryafis and Papadopoulos studied similar pricing problems under the fuzzy environment and utilizing fuzzy numbers (Guerra et al., 2011; Chryafis and Papadopoulos, 2009). Few researchers have explored binary option valuation under fuzzy environments, however. In this paper, we introduce a correlative definition of fuzzy numbers and their operations; we also derive a fuzzy pricing model of the cash-or-nothing binary European call option.

The remainder of this paper is organized as follows. The pricing model of cash-or-nothing binary option under a stochastic environment is derived in Section 2. In Section 3, we introduce the basic characteristics of fuzzy numbers as utilized throughout this paper. A general fuzzy pricing model of binary options is discussed in Section 4, and the numerical experiments we conducted to verify the model’s effectiveness are discussed in Section 5. Section 6 provides a brief summary and conclusion.

2. PRICING MODEL OF CASH-OR-NOTHING BINARY OPTION UNDER STOCHASTIC ENVIRONMENT

There have been many notable breakthroughs in option pricing theory, namely in 1973, when Black and Scholes established the model for valuing European options. Their pricing method can be used to valuate any claim in the Black-Scholes model:

\[ dS_t = rS_t dt + \sigma S_t dW_t, \quad 0 \leq t \leq T, \quad S_0 = S \]  

So

\[ S_t = S_0 e^{\sigma W_t + (r - \sigma^2/2)t} \]  

where \( r \) is the risk-free rate of interest, \( \sigma \) is volatility, and \( W_t \) is the Brownian motion. Let \( T \) be the maturity date and \( [T_0, T] \) denote the final time interval; let \( K \) denote the strike price and \( V(S_t, t) \) denote the price of the binary call option at time \( t \). The pricing model of the cash-or-nothing European call option at time \( t \) can then be written as follows:

\[ \frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0 \]  

with the following boundary condition:

\[ H(S_T, T) = \begin{cases} Q & S_T > K \\ 0 & S_T \leq K \end{cases} \]  

where \( K \) denotes the strike price and \( Q \) is a constant.

To solve Eqs. (3) and (4) for the cash-or-nothing binary option, let

\[ \tau = T - t, \quad x = \ln \frac{S}{K} \]  

based on upper transformation, because

\[ H(S - K) = H\left(\frac{S}{K} - 1\right) = H(e^x - 1) = H(x) \]  

Thus, Eqs. (4) and (5) can be translated into the following Cauchy problem:


\[
\frac{\partial V}{\partial r} = \frac{\sigma^2}{2} \frac{\partial^2 V}{\partial x^2} + (r - \frac{\sigma^2}{2}) \frac{\partial V}{\partial x} - rV = 0
\]

\[
V(x,0) = H(x)
\]

Similar to the derivation of the Black-Scholes formula, we have

\[
V(x,\tau) = e^{-r\tau}N\left(\frac{x + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}\right)
\]

where \( N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{w^2}{2}} dw \). Because \( x = \ln \frac{S}{K}, \tau = T - t \), we have

\[
V(S,t) = e^{r(T-t)}Q(d)
\]

where \( d = \frac{\ln \frac{S}{K} + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \).

3. FUZZY NUMBER

This section provides several correlative definitions and operations of fuzzy numbers that will be utilized throughout this paper (Zadeh, 1965; Zimmermann, 2001).

3.1 Correlative Definition

Definition 1. A fuzzy set \( A \in X \subset R \), where \( R \) is the set of real numbers, is a set of ordered pairs \( A = \{(x, \mu(x)) : x \in X\} \), where \( \mu(x) \) is the membership function of \( x \in X \) which maps \( x \in X \) onto the real interval \([0,1]\).

Let \( R \) denote a universal set of all real numbers. Then a fuzzy subset \( \tilde{A} \) is defined by its membership function \( \tilde{A} : R \rightarrow [0,1] \). The \( \alpha \) – level set of \( \tilde{A} \) is defined by \( \tilde{A}_\alpha = \{x \mid \mu_x(x) \geq \alpha\}, \ (0 \leq \alpha \leq 1) \) The 0-level set \( \tilde{A}_0 \) of \( \tilde{A} \) is defined by the closure of the set \( \{x \mid \mu_x(x) \geq 0\} \), which is called a “normal fuzzy set” if there exists a \( x \) such that.

Definition 2. A fuzzy set \( \tilde{A}_\alpha \) in \( R^\alpha \) is called a “convex fuzzy set”, if and only if for any \( x_1, x_2 \in R^\alpha \) and \( 0 \leq \alpha \leq 1 \),

\[
\mu_{\tilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}
\]

Definition 3. The following conditions must be satisfied for \( \tilde{A} \) to be defined as a fuzzy number:

(1) \( \tilde{A} \) is a normal and convex fuzzy set;
(2) Its membership function \( \mu_{\tilde{A}} \) is upper semi-continuous;
(3) The \( \alpha \) – level set \( \tilde{A}_\alpha \) is bounded for all \( \alpha \in [0,1] \).

Zadeh proved that if \( \tilde{A} \) is a fuzzy number, then \( \tilde{A}_\alpha \) is a convex and compact set (Zadeh, 1996). That is, \( \tilde{A}_\alpha \) is a closed interval, denoted by \( \tilde{A}_\alpha = [\tilde{A}_\alpha^L, \tilde{A}_\alpha^U] \) and \( \tilde{A}_\alpha \) has the following property:

\[
\alpha < \alpha' \Rightarrow \tilde{A}_\alpha \subseteq \tilde{A}_{\alpha'}, \tilde{A} \subseteq \tilde{B} \iff \tilde{A}_\alpha \subseteq \tilde{B}_\alpha, \forall \alpha \in [0,1]
\]

The fuzzy set and membership function are an extension of the classical set and characteristic function. A crisp or usual number is introduced accordingly.

If the membership function of \( \tilde{A} \) is in the following format:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
1, & x = m \\
0, & x \neq m
\end{cases}
\]

then \( \tilde{A} \) is a crisp or usual number that is denoted by \( \tilde{A} = \overline{1_m} \). It readily follows that \( \overline{1_m} = \overline{1_m} \), \( m \in [0,1] \), and that any real number can be considered a crisp or usual number.
**Definition 4.** If the membership function of \( \tilde{A} \) has the following form:

\[
\mu_A(x) = \begin{cases} 
1 - \frac{a-x}{\gamma} & a - \gamma \leq x \leq a, \gamma > 0 \\
1 & a \leq x \leq b \\
1 - \frac{x-b}{\beta} & b \leq x \leq b + \beta, \beta > 0 \\
0 & \text{otherwise}
\end{cases}
\]

(13)

then the fuzzy number \( \tilde{A} \) is a “trapezoidal fuzzy number”, which has the core \([a, b]\), left width \( \gamma \), and right width \( \beta \).

Let \( \tilde{A} = (a, b, \gamma, \beta) \) denote a trapezoidal fuzzy number, then it follows that the \( \alpha \)-level sets of \( \tilde{A} \) have the following form:

\[
\tilde{A}_\alpha = [\tilde{A}_\alpha^-, \tilde{A}_\alpha^+] = [a - (1-\alpha)\gamma, b - (1-\alpha)\beta] \quad \forall \alpha \in [0, 1]
\]

(14)

### 3.2 Fuzzy Number Operation

If the binary operation of two fuzzy numbers \( \tilde{A}, \tilde{B} \) is defined, then the membership function of \( \tilde{A}, \tilde{B} \) is as follows:

\[
\mu_{\tilde{A}, \tilde{B}}(z) = \sup_{(t,r)\in z} \min\{\mu_A(x), \mu_B(x)\}
\]

(15)

where “\( \circ \)” denotes “+” or “\( \times \)” operation, and can be easily expanded to “-” and “/” operation.

Let \( \tilde{A} \) and \( \tilde{B} \) be two fuzzy numbers. Thus, \( \tilde{A} = [\tilde{A}_a^-, \tilde{A}_a^+] \) and \( \tilde{B} = [\tilde{B}_a^-, \tilde{B}_a^+] \); thus \( \tilde{A} \circ \tilde{B}, \tilde{A} \circ \tilde{B} \), and \( \tilde{A} \otimes \tilde{B} \) are also fuzzy numbers with \( \alpha \)-level sets in the following format:

\[
(\tilde{A} \circ \tilde{B})_\alpha = [\tilde{A}_\alpha^-, \tilde{A}_\alpha^+] = [\tilde{A}_a^- \circ \tilde{B}_a^+, \tilde{A}_a^+ \circ \tilde{B}_a^-],
\]

\[
(\tilde{A} \otimes \tilde{B})_\alpha = [\tilde{A}_\alpha^- \circ \tilde{B}_\alpha^+, \tilde{A}_\alpha^+ \circ \tilde{B}_\alpha^-],
\]

\[
(\tilde{A} \circ \tilde{B})_\alpha = [\min(\tilde{A}_\alpha^-, \tilde{B}_\alpha^+), \max(\tilde{A}_\alpha^+, \tilde{B}_\alpha^-)], \quad \forall \alpha \in [0, 1].
\]

If \( \tilde{B}_\alpha \) denotes the \( \alpha \)-level set of \( \tilde{B} \) without zero, then \( \tilde{A} \circ \tilde{B} \) is also a fuzzy number with the \( \alpha \)-level set as the following:

\[
(\tilde{A} \circ \tilde{B})_\alpha = [\min(\frac{\tilde{A}_\alpha^-}{\tilde{B}_\alpha^+}, \frac{\tilde{A}_\alpha^+}{\tilde{B}_\alpha^-}), \frac{\max(\tilde{A}_\alpha^-, \tilde{B}_\alpha^+)}{\max(\tilde{A}_\alpha^+, \tilde{B}_\alpha^-)}], \quad \forall \alpha \in [0, 1].
\]

### 4. Fuzzy Pricing Model of European Call Binary Option

Based on the assumption that the underlying assets price \( \tilde{S} \), interest rate \( \tilde{r} \), and volatility \( \tilde{\sigma} \) are all fuzzy numbers and under the operational principle of fuzzy numbers, the fuzzy pattern pricing model of the cash-or-nothing European call binary option (Eq. (10)) is as follows:

\[
\tilde{V}(\tilde{S}, t, K, \tilde{r}, \tilde{\sigma}) = (e^{-\tilde{r}\tilde{I}_{t<K}}) \otimes \tilde{N}(\tilde{d})
\]

(16)

where \( \tilde{d} = ((\ln(\tilde{S}/\tilde{I}_{t<K})) + (\tilde{r} - \tilde{I}_{t<K} \otimes \tilde{\sigma}^2) \otimes \tilde{I}_{t<K}) / (\tilde{\sigma} \otimes \sqrt{\tilde{I}_{t<K}})) \).

The underlying assets price \( K \) and the time \( t \) are both real (usual) numbers, which are denoted by the crisp numbers \( \tilde{I}_{[k]} \) and \( \tilde{I}_{[t]} \) with values \( K \) and \( t \), respectively, so the fuzzy pricing model of a cash-or-nothing European call binary option at time \( t \) is:

\[
\tilde{C}_t = \tilde{V}(\tilde{S}, t-t, K, \tilde{r}, \tilde{\sigma})
\]

(17)

where \( \tilde{C}_t \) and \( \tilde{S}_t \) are fuzzy random variables and \( \forall t \in [0, 1] \).

The function \( N(x) \) is an increasing function, so the \( \alpha \)-level set of \( \tilde{N}(\tilde{d}) \) is written as follows:
\[(N(\tilde{d}))_a = \{N(x) | x \in \tilde{d}_a \} = \{N(x) | \tilde{d}_a \leq x \leq \tilde{d}_a \} = [N(\tilde{d}_a), N(\tilde{d}_a)] \] (18)

Similarly, the function \(e^{-x}\) is a decreasing function of \(x\) and the function \(\ln x\) is an increasing function of \(x\), so \(e^{-\beta_{l,x}}\) and \(\ln(\tilde{S}/\tilde{I}_{(K)})\) have the following \(\alpha\) – level sets:

\[(e^{-\beta_{l,x}})_a = [e^{-\beta_{l,x}(T-t)}, e^{-\beta_{l,x}(T-t)}] \] (19)

\[(\ln(\tilde{S}/\tilde{I}_{(K)}))_a = [\ln(\tilde{S}_a/K), \ln(\tilde{S}_a/K)] \] (20)

The right-end and left-points of the closed interval \((\tilde{C})_a = [(\tilde{C})_a, (\tilde{C})_a]\) can now be determined, which makes it easy to calculate the right-end point \((\tilde{C})_a^+\) and the left-end point \((\tilde{C})_a^-\) of the closed interval \((\tilde{C})_a\) as follows:

\[(\tilde{C})_a^+ = e^{-\omega(T-t)} N(\tilde{d}_a^-) \] (21)

\[(\tilde{C})_a^- = e^{-\omega(T-t)} N(\tilde{d}_a^+) \] (22)

where

\[\tilde{d}_a^- = \frac{\ln(\tilde{S}_a^-/K) + (\tilde{r}_a^- - (1/2)(\tilde{\sigma}_a^-)^2)(T-t)}{\tilde{\sigma}_a^- \sqrt{T-t}} \quad \forall \alpha \in [0,1] \]

\[\tilde{d}_a^+ = \frac{\ln(\tilde{S}_a^+/K) + (\tilde{r}_a^+ - (1/2)(\tilde{\sigma}_a^+)^2)(T-t)}{\tilde{\sigma}_a^+ \sqrt{T-t}} \quad \forall \alpha \in [0,1] \]

If the underlying assets price \(\tilde{S}\), risk-free rate of interest \(\tilde{r}\), and volatility \(\tilde{\sigma}\) are trapezoidal fuzzy numbers, the specific fuzzy pricing model of the cash-or-nothing European call option can be derived accurately. If \(\tilde{S}\), \(\tilde{r}\), and \(\tilde{\sigma}\) are all trapezoidal fuzzy numbers and \(\tilde{S} = (S_1, S_2, \gamma_S, \beta_S)\), \(\tilde{r} = (r_1, r_2, \gamma_r, \beta_r)\), \(\tilde{\sigma} = (\sigma_1, \sigma_2, \gamma_\sigma, \beta_\sigma)\), then their \(\alpha\) – level set is as follows:

\[\tilde{S}_a = [S_1 - (1-\alpha)\gamma_S, S_2 + (1-\alpha)\beta_S] \]

\[\tilde{r}_a = [r_1 - (1-\alpha)\gamma_r, r_2 + (1-\alpha)\beta_r] \]

\[\tilde{\sigma}_a = [\sigma_1 - (1-\alpha)\gamma_\sigma, \sigma_2 + (1-\alpha)\beta_\sigma] \]

The fuzzy pricing formula of \(\tilde{C}\) can be obtained by using the fuzzy pattern of the cash-or-nothing call binary option pricing model (Eq. (17)). The \(\alpha\) – level set of the cash-or-nothing call binary option price \(\tilde{C}_a\) can be written in crisp form as follows:

\[(\tilde{C})_a = [(\tilde{C})_a^-, (\tilde{C})_a^+], \forall \alpha \in [0,1] \] (23)

\[(\tilde{C})_a^+ = e^{-(r_2-(1-\alpha)\gamma_r)(T-t)} N(\tilde{d}_a^-) \] (24)

\[(\tilde{C})_a^- = e^{-(r_1-(1-\alpha)\gamma_r)(T-t)} N(\tilde{d}_a^+) \] (25)

where

\[\tilde{d}_a^- = \frac{\ln((S_1 - (1-\alpha)\gamma_S)/K) + (r_1 - (1-\alpha)\gamma_r - (1/2)(\sigma_1 - (1-\alpha)\gamma_\sigma)^2)(T-t)}{\sigma_1 - (1-\alpha)\gamma_\sigma \sqrt{T-t}} \]

\[\tilde{d}_a^+ = \frac{\ln((S_2 + (1-\alpha)\beta_S)/K) + (r_2 + (1-\alpha)\beta_r - (1/2)(\sigma_2 + (1-\alpha)\beta_\sigma)^2)(T-t)}{\sigma_2 + (1-\alpha)\beta_\sigma \sqrt{T-t}} \]

If \(\gamma_S = \gamma_r = 0\) and \(\beta_S = \beta_r = 0\), then \(\tilde{S}, \tilde{r},\) and \(\tilde{\sigma}\) are all interval numbers.

The fuzzy pricing formula of \(\tilde{C}\) can be obtained by using the fuzzy pattern of the cash-or-nothing call binary option pricing model (Eq. (23)). The cash-or-nothing call binary option price \(\tilde{C}_a\) can be given by the following interval:

\[\tilde{C}_a = [C_a^-, C_a^+] \] (26)

where
5. NUMERICAL EXPERIMENTS

Consider the European call binary option with the underlying assets price $K=30$ and $T=6$ (month). Assume that the underlying assets price $S=35$, $Q=10$, the volatility of underlying assets $\sigma=0.2$, and the risk-free rate of interest $r=0.05$ (per annum). Suppose that $t=0$ and $T=0.5$. The fuzzy underlying assets price $\tilde{S}_0$, fuzzy risk-free rate of interest $\tilde{r}$, and fuzzy volatility of underlying assets price $\tilde{\sigma}$ are all assumed as trapezoidal fuzzy numbers: $\tilde{S}_0 = (34.7, 35.2, 1.9, 2.6)$, $\tilde{r} = (0.047, 0.052, 0.012, 0.014)$, and $\tilde{\sigma} = (0.18, 0.22, 0.05, 0.06)$, respectively. According to the fuzzy option pricing model derived above, the current fuzzy price $\tilde{C}_0$ of the cash-or-nothing call binary option can be obtained accurately.

Given different the cash-or-nothing call binary option prices $c$, using the algorithm derived by Wu (Wu, 2005), the belief degrees $\mu_{\tilde{C}_0}(c)$ are given in Table 1.

<table>
<thead>
<tr>
<th>$C$</th>
<th>$\mu_{\tilde{C}_0}(c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.63</td>
<td>0.8271</td>
</tr>
<tr>
<td>3.72</td>
<td>0.9025</td>
</tr>
<tr>
<td>3.81</td>
<td>0.9520</td>
</tr>
<tr>
<td>3.90</td>
<td>0.9859</td>
</tr>
<tr>
<td>4.22</td>
<td>1.0000</td>
</tr>
<tr>
<td>4.52</td>
<td>0.9732</td>
</tr>
<tr>
<td>4.63</td>
<td>0.9455</td>
</tr>
<tr>
<td>4.72</td>
<td>0.8913</td>
</tr>
<tr>
<td>4.85</td>
<td>0.8246</td>
</tr>
</tbody>
</table>

Table 1 yields a few interesting conclusions. If the cash-or-nothing call binary option price is 3.81, its belief degree is 0.9520. Accordingly, if the investor is satisfied with the belief degree 0.9520, then he or she can take this option for the price of 3.81 for use in the future. If the cash-or-nothing call binary option price is 4.22, then its belief degree equals 1.00. This situation reflects the fact that the cash-or-nothing call binary option calculated via the traditional formula is 4.22 (based on $r=0.05$, $\sigma=0.2$, $S_0=35$).
Table 2. α- level closed intervals of the cash-or-nothing call binary option fuzzy price

<table>
<thead>
<tr>
<th>α</th>
<th>((\tilde{C}_0))_α</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>[3.6975, 4.7424]</td>
</tr>
<tr>
<td>0.92</td>
<td>[3.7463, 4.6938]</td>
</tr>
<tr>
<td>0.94</td>
<td>[3.7948, 4.6453]</td>
</tr>
<tr>
<td>0.96</td>
<td>[3.8434, 4.5967]</td>
</tr>
<tr>
<td>0.98</td>
<td>[3.8919, 4.5481]</td>
</tr>
<tr>
<td>1.00</td>
<td>[3.9405, 4.4995]</td>
</tr>
</tbody>
</table>

Table 2 shows the α – level closed intervals (\(\tilde{C}_0\))_α of the cash-or-nothing call binary option fuzzy price. If assuming α=0.96, then (\(\tilde{C}_0\))_α = [3.8434, 4.5967] – this means that the option price will lie in [3.8434, 4.5967] when belief degree α =0.96. It also means that if the investor is interested at belief degree 0.96, then he or she may take any value from the interval [3.8434, 4.5967] as the option price for use in the future.

6. CONCLUSION

This paper presented the results of our study on a novel cash-or-nothing call binary option pricing method based on fuzzy number theory. We first introduced some basic concepts and operations related to fuzzy numbers, then established a fuzzy pattern option pricing model. The proposed model was proven feasible and effective via two numerical experiments.

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REFERENCES


