**Investment Portfolio Selection Model Based on Typical Transaction Cost**

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**Abstract**  
This paper attempts to build an investment portfolio optimization model based on the typical transaction cost. First, shortages of current investment portfolio models are analyzed. By analyzing the changing process of the transaction cost in practical investments, the author introduces the non-concave and non-convex transaction cost function and the mean-variance model, and adopts the investment risk value as the objective value and the comprehensive earnings value as the restriction under the prerequisite of no short selling. The rate of return on investment, the rate of investment dividends and the typical transaction cost function are taken into consideration to build the investment portfolio optimization model. At last, the improved PSO algorithm is adopted to get solutions. Based on cases of numerical value, the influence of different transaction cost functions and model parameters on the investment portfolio is analyzed. Calculation results suggest that: the typical transaction function can achieve an efficient investment portfolio and increase the market investment efficiency. The weight coefficient, $\beta$, of objective risks, does not influence the objective investment portfolio results, but just influence the risk value, $f$. The judgment of the risk value is not influenced.

**Key words:** Investment Portfolio, Transaction Cost, Mean Value-Variance, Improved PSO

1. INTRODUCTION  
Optimization based on the expected objective has found wide applications in the field of economic planning, production management and engineering design (Jiangze, 2014). The investment portfolio in Finance is an important research field of objective optimization (Yiping, 2013; Jinho, Kang and Park, 2014). Investment portfolio means distribution of resources of certain cost to different investment projects so as to minimize the risks under the prerequisite of meeting the expected return rate (Tao & Ling, 2014; Hwang, Gao and Owen, 2014). Markowitz first put forward the idea of using variance to measure risks facing stock earnings (Hwang T, Gao and Owen, 2014) in 1952. Through establishment of multi-objective optimization problem (MOOP), he built the mean-variance analysis model for investment portfolio selection. The model directly transforms economic phenomena into mathematical optimization issues. It lays the foundation for financial research.

However, previous researches into investment portfolios all attempted to simplify the model. The model thus built just takes into consideration the earnings and risks of assets, while ignoring other influencing factors during the investment period, such as transaction cost, tax revenue and dividend. A model without comprehensive consideration of all possible conditions might come up with invalid investment portfolio optimization results. These results might differ greatly from the actual results. Therefore, how to improve and optimize the current investment portfolio selection model and decrease the error between the model results and the actual results is an issue of great concern.

Currently, some modified models have been put forward. For example, some introduced linear and nonlinear transaction cost functions to the investment portfolio selection process (Yuting, 2014); some modified the model with the V-type transaction cost model (Mei, 2013). However, the actual transaction cost is a non-convex and non-concave function with two turning points of the asset transaction volume (Zengxin, Xin & Zhichao, 2013). Therefore, all the above transaction cost functions vary greatly from the practical situations. Concerning the problem, this paper introduces the typical transaction cost function based on modification of the transaction cost function model, and considers the influence of dividends. An investment portfolio optimization model based on the typical transaction cost is built up. The improved PSO algorithm is adopted to solve the model. At last, the author comes to the conclusion that the typical transaction function can help achieve a relatively efficient asset portfolio and get closer to the practical situations.

2. INVESTMENT PORTFOLIO OPTIMIZATION MODEL  
2.1. Typical transaction cost function model  
At present, many investment portfolio models have ignored the influence of many other factors for the sake of simplification. During the practical process, the transaction cost has a huge influence on the practical investment portfolio. Lots of researches have suggested that the practical transaction cost function is a non-
convex and non-concave function with two turning points of the asset transaction volume. Therefore, linear, nonlinear and V-type functions all vary greatly from the practical situations. According to the practical transaction cost function, when the asset amount is small, the unit trade cost is huge; when the trade amount increases, the unit transaction cost will stabilize gradually; when the asset amount is huge, the unit transaction cost will increase dramatically in the end. Changes of the typical transaction cost function are shown in Fig. 1 below. When the asset amount is small and has not yet reached Point A, the transaction cost function, C(x), is in the shape of a concave. After passing point, the unit transaction cost maintains relatively steady. In other words, the transaction cost function, C(x), show linear increase along with the increase of the trade volume. Before the turning point of B, if the transaction volume keeps on increasing, the lack of stock supply will result in dramatic increase of the transaction cost. Therefore, the cost function, C(x), will turn into the shape of a convex after passing Point B.

The above typical transaction cost function, C(x), can be expressed as Eq. 1 below:

\[
C(x) = \begin{cases} 
\lambda \sqrt{x} & 0 \leq x < a \\
\lambda (kx + h) & a \leq x \leq b \\
\lambda (x^2 + m) & b < x \leq 1 \\
\lambda > 0 
\end{cases}
\]

Generally speaking, in order to guarantee continuity of the typical cost function, C(x), when \( k \cdot a \) and \( b \) are fixed, \( h = \sqrt{a} - ka = kb + h - b^2 \).

2.2. Establishment of the investment portfolio model based on the typical transaction cost

In order to make the model results approximate to practical situations, this paper takes the influence of earnings, dividends and typical transaction cost of investment portfolios on security investment into consideration while building the investment portfolio model.

It is assumed that there are \( n \) kinds of securities in the market; \( r_i \) stands for the rate of earnings of the \( i \) kind of security; \( r_2 \) stands for the rate of dividends of the \( i \) kind of security; \( x_i \) stands for the investment proportion of the \( i \) kind of security; \( C_i(x_i) \) stands for the transaction cost of the \( i \) security; \( \sigma_{ai} \) stands for the covariance coefficient of the earnings rate between the \( i \) kind of security and the \( j \) kind of security; \( \sigma_{a2} \) stands for the covariance coefficient of the dividend rate between the \( i \) kind of security and the \( j \) kind of security; \( g_1 \) stands for investment gains; \( g_2 \) stands for investment gains of dividends; \( a_1 \) stands for the objective value of investment earnings; \( a_2 \) stands for the objective value of dividend earnings; \( f_1 \) stands for the risk value of investment earnings; \( f_2 \) stands for the risk value of investment dividends; \( f \) stands for the comprehensive risk value of the investment portfolio.

Here, based on the mean-variance model of Markowitz and under the prerequisite of no short selling, the author adopts the minimum risk value as the objective and the comprehensive earnings value as the restriction to build the following investment portfolio model (See Eq. 2):
\[
\min f_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}
\]
\[
\min f_2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}
\]
\[
g_1 = \sum_{i=1}^{N} r_i x_i - \sum_{i=1}^{N} C_i (x_j) > a_i
\]
\[
g_2 = \sum_{i=1}^{N} r_i x_i > a_2
\]
\[
\sum_{i=1}^{N} x_i = 1
\]
\[
\text{s.t.}
\]
\[
C_i (x_j) = \begin{cases} 
\lambda \sqrt{x_i} & 0 \leq x_i < a \\
\lambda (kx_i + h) & a \leq x_i \leq b \\
\lambda (x_i^2 + m) & b < x_i \leq 1 \\
\lambda > 0
\end{cases}
\]
(2)

In order to facilitate model solution, the above multi-objective planning can be transformed into the single-objective planning. Then, the comprehensive objective function can be obtained, namely
\[
\min f = \beta f_1 + (1 - \beta) f_2
\]
where \(\beta\) stands for the weight coefficient.

3 MODEL SOLUTION BASED ON THE IMPROVED PSO ALGORITHM

3.1 Standard PSO algorithm

This paper puts forward the idea of solving the above investment portfolio optimization models through the improved PSO algorithm (Wei, Runtong & Ling, 2009; Xia, Hongxia & Kejun, 2006). The PSO algorithm is a heuristic optimization calculation model. It assumes that there is a particle swarm which can contain M particles; the number of dimensions of the searching space for the particle swarm is D; and the status attribute value of Particle i at the t moment is shown in Eq. 6 to Eq. 8, respectively:

1: Position status:
\[
X_i = (X_i^1, X_i^2, X_i^3, \ldots, X_i^d)^T
\]
\[
X_{id} \in (X_{min}, X_{max})
\]
(3)

Where, \(X_{min}\) stands for the lower limit of the coordinate position; \(X_{max}\) for the upper limit of the coordinate position.

2: Velocity status:
\[
V_i = (V_i^1, V_i^2, V_i^3, \ldots, V_i^d)^T
\]
\[
V_{id} \in (V_{min}, V_{max})
\]
(4)

Where, \(V_{min}\) stands for the lower limit of the velocity; \(V_{max}\) for the upper limit of the velocity.

3: Individual optimal position:
\[
P_i^* = (P_i^{1*}, P_i^{2*}, P_i^{3*}, \ldots, P_i^{d*})^T
\]
(5)

4: Global optimal position:
\[
P_{gd}^* = (P_{gd}^{1*}, P_{gd}^{2*}, P_{gd}^{3*}, \ldots, P_{gd}^{d*})^T
\]
(6)

All the above is the status attribute value of the particle at the t moment; while the status property of the particle at the “t+1” moment can be updated and iterated through Eq. 7.

\[
V_{id}^{t+1} = w V_{id}^t + c_1 r_1 (P_i^{d*} - X_i^t) + c_2 r_2 (P_{gd}^{d*} - X_i^t)
\]
\[
X_{id}^{t+1} = X_{id}^t + V_{id}^{t+1}
\]
(7)

Where, \(w\) stands for the inertia weight value of the PSO algorithm; \(c_1, c_2\) for the acceleration constants of the PSO algorithm; \(r_1, r_2\) for the random variables and obey the even distribution within the region (0, 1)
When the status attribute value of the particle at the \( t \) moment is iterated into that at the \( t+1 \) moment, there contain three core parts. First, the velocity of the particle of the former moment, meaning that the velocity of the particle in the latter moment keeps the velocity in the former moment; second, self-cognition, meaning that the particle will take its flying experiences into consideration and adjust its flying state according to the historical optimal position; third, the social factor, meaning that the particle will also refer to the flying information of the whole population apart from referring to its own experiences, and adjust its flying state according to the swarm’s optimal position.

3.2. Improved PSO algorithm

3.2.1 Optimization of the PSO algorithm based on linear declination of velocity iteration factors

Based on the above standard PSO, it can be seen that, during the iteration process, \( w \) stands for the inertial weight value of the PSO algorithm. The higher the weight value is, the stronger the global searching capability is. When the weight value is relatively small, it has a relatively strong local searching capability. Therefore, in the standard PSO algorithm, \( w \) is set at a fixed value, and its algorithm convergence is relatively poor, which might fail to get the optimal solution. Therefore, in order to improve the convergence of the standard PSO algorithm and endow the algorithm with a good global searching capability in the early operation period and a good local searching capability in the latter operation period, \( w \) gradually changes with the model’s iteration number. The iteration equation of \( w \) is shown in Eq. 8 below:

\[
w(t) = w_2 + (w_1 - w_2) \frac{T-t}{T}
\]

Where, \( w_1 \) stands for the initial inertial weight value; \( w_2 \) for the inertial weight value at the end; \( t \) for the iteration time; \( T \) for the maximum iterations.

Besides, from the standard PSO, it can be seen that, when \( c_1, c_2 \) are relatively small, the particle can partially adjusted within the optimal objective region; when \( c_1, c_2 \) are relatively large, particles far away can move quickly to the objective region. Therefore, proper adjustment of the value of \( c_1, c_2 \), can help the model stay close to the optimal value more easily. The iteration equation for the acceleration constants, \( c_1, c_2 \), is shown in Eq. 9:

\[
c_1(t) = c_{1i} + (c_{1f} - c_{1i}) \frac{t}{T}
\]

\[
c_2(t) = c_{2i} + (c_{2f} - c_{2i}) \frac{t}{T}
\]

Where, \( c_{1i} \) and \( c_{2i} \) are initial acceleration constants; \( c_{1f} \) and \( c_{2f} \) are final acceleration constants; \( t \) is the iteration number; and \( T \) is the maximum iteration number.

To sum up, the PSO velocity and position update equation based on the linear declination of the velocity iteration is shown below: (See Eq. 10)

\[
V_{id}^{t+1} = w(t)V_{id}^t + c_1(t)r_1(P_{id}^t - X_{id}^t) + c_2(t)r_2(P_{gd}^t - X_{id}^t)
\]

\[
X_{id}^{t+1} = X_{id}^t + V_{id}^{t+1}
\]

3.2.2 Optimization of the PSO algorithm based on the self-adaptive variation

From the standard PSO, it can be seen that all particles stay close to the optimal position, \( P_g \). When the optimal position becomes a local optimal point, the standard PSO cannot re-search in the solution space and becomes trapped in the local optimal solution (Deyi, Zhengyou and Zang, 2011). Therefore, it is necessary to conduct variation operation of the global optimal position, \( P_g \), change the forward direction of parameters during the iteration process, and enter the other regions to keep on searching for the global optimal solution. Certain dimension is randomly chosen for the variation operation. The variation probability, \( p \), is shown in Eq. 11 below:

\[
p = \begin{cases} 
  k & \sigma_i^2 < \sigma_d^2 \cdot f(P_g^t) > f_d \\
  0 & \text{otherwise}
\end{cases}
\]
Where, \( k \) is a random value within the section of \([0.1, 0.3]\); \( \sigma^2 \) for the group fitness variance; \( f_d \) for the optimal objective solution. The particle’s position variation is shown in Fig. 15 below, where \( \eta \) is a random variable within the section of \( \text{gauss}(0, 1) \).

\[
P'_t = (P'_{t1}, P'_{t2}, P'_{t3}, \ldots, P'_{td})^T
\]

\[
P'_{ti} = P'_t(1 + 0.5\eta), i = 1, 2, \ldots, d
\]  

(12)

4. NUMERICAL SIMULATION

In order to directly describe the model optimization results, this paper assumes parameters to be calculated by the model are as follows. There are three kinds of securities involved in the model hypothesis. Their expected return rate is \( r_1 = 0.61 \), \( r_2 = 0.15 \) and \( r_3 = 0.5 \), respectively; their dividend rate is \( r_{d1} = 0.05 \), \( r_{d2} = 0.018 \) and \( r_{d3} = 0.09 \). In the typical cost function, \( \lambda = 0.05 \), \( a = 0.3 \), \( b = 0.7 \) and \( k = 2 \). In the objective function, \( \beta = 0.3 \).

\[
\sigma_{a1} = \begin{bmatrix} 0.025 & 0.015 & 0.017 \\ 0.015 & 0.021 & 0.009 \\ 0.017 & 0.009 & 0.001 \end{bmatrix}, \sigma_{a2} = \begin{bmatrix} 0.141 & -0.189 & 0.167 \\ -0.189 & 0.260 & -0.220 \\ 0.167 & -0.220 & 0.224 \end{bmatrix}
\]

In order to compare the influence of different transaction cost functions on optimization of investment portfolios, the author set the objective value of the dividend earnings to be \( a_1 = 0.02 \); adopts the objective value of investment earnings, \( a_1 \), as the independent variable, and the comprehensive risk objective value, \( f \), as the dependent variable to calculate the optimal solution of investment portfolios. The solution results are shown in Fig. 2 below:

![Figure 2. Changes of the risk value under different investment earnings objectives](image)

Based on solution results shown in Fig. 2, it can be seen that: in terms of any transaction cost function, when the objective earnings value increases, the corresponding comprehensive risk value will increase as well. Without considering the cost transaction, the risk value is the minimal. When the V-type cost function is taken into consideration, the risk value is the maximal. However, the risk value is between the minimal and the maximal when the typical transaction cost function is considered. In other words, when the objective earning value is fixed, the risk value given by the V-type cost function is relatively high, which might cause misunderstanding of high risks and missing of an investment opportunity. The risk value is relatively low when the transaction cost is not considered. Under the consideration, investors might think that the objective earnings of the current investment is too low, thus vigorously increasing their investment cost, which might result in greater losses. Therefore, if the transaction cost model is ignored, the investment portfolio selection might be invalid. When the typical transaction cost model is considered, the potential earnings will not be overestimated under fixed risk value, and the potential risks will not be overestimated under fixed earnings. Therefore, the typical transaction cost can achieve a more efficient asset portfolio.
In order to deeply analyze the influence of model parameters on the typical transaction cost function model and to test the sensitivity of the model parameters, parameters, including $\beta$, $a_i$, and $x_i$, are transformed, while the other parameters remain the same so as to test the influence of parameter changes on model results and to obtain rules about the optimal asset portfolio, expected earnings and practical risk value. The results are shown in Table 1 above:

Based on model optimization results of the above parameters, with the increase of the objective investment earnings, $a_i$, and the objective dividends, $f$, the comprehensive risk value, $\beta$, will increase correspondingly. When the weight coefficient $\beta$ increases, the optimal investment portfolio results, namely $x_i$, $x_2$ and $x_3$, are not changed. In other words, changes of the weight coefficient $\beta$ will not influence the optimal objective investment portfolio results. However, when the weight coefficient $\beta$ increases, the comprehensive risk value, $f$, will be on a mild downward trend. The result is influenced by the covariance matrix of the earnings rate and the dividend rate of three securities. In other words, under different weight proportion, the accumulated results of the covariance matrix of the earnings rate and the dividend rate of three securities will differ slightly. However, the general fluctuation is not huge, so the risk value judgment will not be influenced.

### 5. CONCLUSIONS

This paper succeeds in building an investment portfolio optimization model based on the typical transaction cost function model in line with the practical investment process is put forward. Based on the mean-variance model of Markowitz and under the prerequisite of no short selling, the investment risk value is adopted as the objective value and the comprehensive earnings value is adopted as the restriction to build the investment portfolio optimization model fully considering the investment earnings rate, the investment dividend rate and the typical transaction cost function. The improved PSO is employed to solve the model. The influence of different transaction cost function structures and model parameters on the investment portfolio optimization model built up in this paper is illustrated. The numerical value results suggest: 1) Different types of transaction costs can influence the efficient frontier of investment portfolios; while the typical transaction cost can help achieve a relatively efficient investment portfolio. The investment portfolio optimization model based on the typical transaction cost in this paper can increase the market investment efficiency; 2) The weight coefficient, $\beta$, will not influence the optimal objective investment portfolio results, but might influence the risk value, $f$, to some extent. However, the judgment of the risk value is not influenced.

### REFERENCES


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**Table 1. Optimization results of investment portfolios under different model parameters**

<table>
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<tr>
<th></th>
<th>$\beta = 0.3$</th>
<th>$\beta = 0.5$</th>
<th>$\beta = 0.7$</th>
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<td>$a_i$</td>
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<td>0.40</td>
<td>0.45</td>
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<td>$a_2$</td>
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<td>0.059</td>
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Mei W. (2013) “Methods to evaluate and apply the investment portfolio techniques considering market friction”, *Thesis of Hunan University*.


