Traffic Strategies of Multiple MNOs: A Quantization Approach

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Abstract
To promote continuous deepening of reforms in the public services areas across China, the Ministry of Industry and Information Technology of the People's Republic China has issued the mobile virtual network operators licenses to 19 private enterprises successively from the end of 2013. The MVNOs will get the official licenses of mobile business this year, making the market more and more open and free. We investigate the quantization of strategies with continuum space for multiple MNOs in China. We model a quantum structure, where each MNO is given by a qubit and entangles pairwise. Through the research, we find the more value of \( n \) leads to the more intense changes of optimal utility with the increase of \( \gamma \). No matter how many MNOs participating in the market, the equilibrium strategic value of optimal utility would increase rapidly as long as the increase of \( \gamma \), even slightly.

Key words: Continuum Space, Equilibrium Strategic Value, Quantization of Strategies

1. INTRODUCTION

Nowadays, people in China are largely in the habit of accessing various sophisticated mobile services from their smart mobiles. When absent from free Wi-Fi hotspots, an end-user has to completely depend upon mobile oligopolistic network operators(MNOs), i.e. China Mobile, China Unicom and China Telecom, who have the monopoly right for their traffic. The growth of mobile traffic is creating the conditions for further technological advancements and service innovations. One brief illustration of available traffic and related services operation is shown by Figure 1, in which the end-users regularly transmit data to the service integrators on demand through MNOs. Once the demand as traffic is processed by the service integrators, the related service as traffic is provided to the end-users. During this process, MNOs help the crowded end-users to access the various mobile services and end-users pay MNOs by the traffic. However, with the increasingly admittances of more MVNOs (i.e., mobile virtual network operators, who have no their own mobile network infrastructures and still face end-users by providing the traffic), the business logic of the market is changing quietly, which is also manifested by Figure 1. To promote continuous deepening of reforms in the public services areas across China, the Ministry of Industry and Information Technology of the People's Republic China has issued the mobile virtual network operators licenses to 19 private enterprises successively from the end of 2013. The MVNOs will get the official licenses of mobile business this year, making the market more and more open and free. The entry of the MVNOs is still more or less a shock for the MNOs in terms of both cooperation and competition, especially in the aspects of available traffic and relative price, even though the MVNOs lease spectrum resources from MNOs. In the future, MVNOs will be dancing in front of the stage, and the MNOs today will be taking care of their mobile network infrastructures behind the scenes devotionally. Of course, it may be a long-term process.

Lots of scholars have attached great importance to the impact of service-based competition and cooperation among MNOs and have analyzed some strategies(Banerjee and Dippon, 2009; Pohjola and Kilkki, 2007). Guijarro et al. modeled a wireless market environment, where a MVNO leases spectrum from an MNO, which is also active in the retail market (Guijarro, Pla and Tuffinet al., 2011). Cadre et al. tackled the problem whether cooperative investment is profitable for both (Cadre and Bouhtou, 2012). In their other article, each provider market share was fixed while they competed on the scarce resource allocation (Cadre, Bouhtou and Tuffin, 2009). Mutlu et al. showed that a deterministic pricing optimizes the operator’s revenue among all the unique price strategies (Mutlu, Alanyali and Starobinski, 2008). Niyato et al. proposed distributed algorithms to achieve the pricing solutions of different pricing models and analyzed stability of distributed algorithms (Niyato and Hossain, 2008). Duan et al. presented a comprehensive analytical study of two competitive secondary operators’ investment (i.e., spectrum leasing) and pricing strategies, taking into account operators’ heterogeneity in leasing costs and users’ heterogeneity in transmission power and channel conditions (Duan, Huang and Shou, 2012).
The literatures above cannot escape the common main modeling approach, such as, mixed integer linear programming model, the exponential distribution or integral model under various parameters, etc. That is, most researches are based on the classical probability theory whose decision space is finite, limited and smaller in service supply chain. However, Quantum mechanics has a bigger probability space and brings a totally new perspective. There are few literatures modeling competition and cooperation through Quantum perspective in service supply chain fields. Quantum theory has created an invigorating buzz for the academic and is attracting new visitors in large. Many other fields like large-scale calculations, nuclear energy and aerospace, etc., have combined with it, resulting in a range of new interdisciplinary sciences.

An increasing number of scholars have been into the interdisciplinary science of quantum strategies since 1999. Meyer considered game theory from the perspective of quantum algorithms, showing the power of quantum strategies for the first time (Meyer, 1999). Eisert et al. introduced an elegant scheme for quantizing classical games, and proceeded to perform an extensive analysis of its application to the famous two-player game, Prisoner’s Dilemma (Eisert, Wilkens and Lewenstein, 1998). After that, Marinatto et al. extended the concept of a classical two-person static game to the quantum domain, by giving a Hilbert structure to the space of classical strategies and studying the Battle of the Sexes game (Marinatto and Weber, 2000). Chen et al. showed that if the handicapped player with classical means can delay his action for a sufficiently long time, the quantum version reverts to the classical zero-sum game under decoherence (Chen, Kwek and Oh, 2002). Frąckiewicz presented the unique solution to the quantum Battle of the Sexes game, showing the best result to be achieved when the game is played according to Marinatto and Weber’s scheme (Frąckiewicz, 2009). Khan et al. presented a quantum model of Bertrand duopoly and studied the entanglement behavior on the profit functions of the firms. Using the concept of optimal response of each firm to the price of the opponent, they found only one Nash equilibrium point for the maximally entangled initial state (Khan and Ramzan, 2010). Frąckiewicz gave a strict mathematical description for a refinement of the Marinatto–Weber quantum game scheme. The model allows the players to choose projector operators that determine the state on which they perform their local operators (Frąckiewicz, 2014). To sum up, a significant aspect of the study of quantum strategies is the exploration of the game theoretic solution concept of the Nash equilibrium in relation to the quantization of a game (Iqbal, Chappell and Abbott, 2015). The above analysis shows that there are few literatures involved more players, especially the impact of service-based competition and cooperation among MNOs. Hence, we intend to introduce quantum theory into the MNOs game in service chain field. By involving entanglement among the states for all the MONs, we investigate a quantum approach to n-MONs game.

2. PROBLEM DESCRIPTION AND ASSUMPTIONS

In spite of MVNOs are not really so powerful in the retail market, MVNOs are increasing their market shares on some foreign markets. We assume that a MONO’s operational cost is equal to the sum of operational cost and the procurement cost of a MVNO, meaning the same total cost for unit traffic to all MNOs. We denote available traffic of the j th MNO as \( t_j \) and the total available traffic as \( T \) (i.e. the total demand is fixed), so \( T = t_1 + t_2 + \cdots + t_n \). Suppose the relative price of available traffic is \( P(T) \). The function of relative price and available traffic is

\[
P(\sum_{j} t_j) = b \cdot \frac{\theta}{n} \sum_{j} t_j + \epsilon
\]
where, as a constant, $\theta$ is an impact factor of the available traffic to the relative price, $\theta > 0$. $\epsilon$ is a usual error term to $P$ and follows a normal distribution such that $\epsilon \sim (0, \sigma^2)$. For convenience, let $k = b - c$, $c$ is the total cost for a MNO. Then, the $j$th MNO’s utility function is

$$u_j = t_j \left[ k - \frac{\theta \sum t_j}{n} + \epsilon \right] \quad (2)$$

The classical Nash equilibrium solutions are as follows:

$$\frac{\partial u_j}{\partial t_j} \left[ k - \frac{\theta \sum t_j}{n} + \epsilon \right] - \frac{\theta}{n} t'_j = 0 \quad (3)$$

So the Nash equilibrium is

$$t'_j = L = t'^n = \frac{nk}{\theta(n+1)} \quad (4)$$

In this condition, the utilities at the equilibrium are

$$u'_j(t'_j, \ldots, t'_n) = L = u'^n(t'_1, \ldots, t'_n) = \frac{nk^2}{\theta(n+1)^2} \quad (5)$$

But this conclusion is not the optimal solution because of completely non-cooperative behaviors, if all MNOs can cooperate completely, we have

$$U = \sum u_j = \left( \sum t_j \right) \left[ k - \frac{\theta \sum t_j}{n} + \epsilon \right] \quad (6)$$

the derivative of $U$

$$U' = k - 2\theta \sum \frac{t_j}{n} = 0 \quad (7)$$

Then the best solution of MNOs will be equal to

$$t^p_j = L = t'^p = \frac{k}{2\theta} \quad (8)$$

The utilities of the MNOs will be

$$u^p_j(\frac{k}{2\theta}, \ldots, \frac{k}{2\theta}) = L = u'^p_n(\frac{k}{2\theta}, \ldots, \frac{k}{2\theta}) = \frac{k^2}{4\theta} \quad (9)$$

This is the Pareto optimum, that is, it is the highest utility they can obtain if they cooperate completely. However, Nash equilibrium fails to be the Pareto optimum because of their selfishness. So, a dilemma-like situation still exists.

3. THE ESTABLISHMENT AND SOLUTION OF THE MODEL

The quantum strategies are on the quantum states, and the utilities can be read out according to the final measurement. A Hilbert space with a continuous set of orthogonal bases is usually set up when quantum domain is introduced into the MNOs game. An unspecified number of MNOs are competing in the market, so we use $n$ single-mode electromagnetic fields. We suppose they have the same entangle degree, i.e. $\gamma = \gamma_1 = \cdots = \gamma_n$. The game starts from the vacuum state, the tensor product, $|\text{vac}_j\rangle \otimes L \otimes |\text{vac}_n\rangle$. This state firstly experiences a symmetric unitary operation known to all MNOs, and after this operation, the state is

$$|\psi\rangle = \hat{J}(\gamma) |\text{vac}_j\rangle \otimes L \otimes |\text{vac}_n\rangle \quad (10)$$

where, the entanglement operator is given by
\[ \hat{J}(\gamma) = \exp \left\{-\gamma \left[ \sum_{j,i,l} \left( \hat{a}_j^* \hat{a}_i - \hat{a}_j \hat{a}_i^* \right) \right] \right\} \]  

(11)

where \( \hat{a}_j^* \) is the annihilation operator of MNO \( j \)'s electromagnetic field.\n
We let the observables of the final measurement be \( \hat{X}_j = (\hat{a}_j^* + \hat{a}_j) / \sqrt{2} \), and “momentum” operators be \( \hat{P}_j = i(\hat{a}_j^* - \hat{a}_j) / \sqrt{2} \), and we have

\[ t_j = \hat{x}_j \]  

(12)

\[ u_j^0 (\hat{S}_1, \ldots, \hat{S}_n) = u_j (\hat{x}_1, \ldots, \hat{x}_n) \]  

(13)

When \( \hat{J}(\gamma) = \hat{I} \) (the identity operators), the quantum strategic space is

\[ S_j = \left\{ \hat{S}_j (x_j) = \exp \left\{-ix_j \hat{P}_j \right\} \mid x_j \in [0, \infty) \right\} \]  

(14)

These strategies are all local unitary operators. We assume that the light beam is infinitely squeezed, thus \( \hat{X}_j \) could be measured precisely. Then, before taking final measurement, it will be carried out by \( \hat{J}^* \) as a disentangling operator. So the final state of model’s quantization before measurement is

\[ \left| \psi_j \right\rangle = \hat{J}(\gamma)^* (\hat{D}_1 \otimes \cdots \otimes \hat{D}_n) \hat{J}(\gamma) \left| \text{vac}_1 \right\rangle \otimes \cdots \otimes \left| \text{vac}_n \right\rangle \]  

(15)

The specific form of the entanglement operator about \( j \) th MNO is

\[ \hat{J}(\gamma)^* \hat{S}_j (x_j) \hat{J}(\gamma) = \exp \left\{-ix_j \left[ \hat{P}_j \frac{1}{\theta} \left( e^{(n-1)r} + (n-1)e^{-r} \right) + \sum_{l \neq j} \hat{P}_l \frac{1}{\theta} \left( e^{(n-1)r} - e^{-r} \right) \right] \right\} \]  

(16)

Then, after taking measurement, we get the traffic of \( j \) th MNO as

\[ t_j = x_j \frac{1}{\theta} \left( e^{(n-1)r} + (n-1)e^{-r} \right) + \sum_{l \neq j} x_l \frac{1}{\theta} \left( e^{(n-1)r} - e^{-r} \right) \]  

(17)

and then,

\[ T = \sum_{j=1}^{n} x_j e^{(n-1)r} \]  

(18)

So, the solution of Nash equilibrium \( \left( \frac{\partial u_j}{\partial t_j} = 0 \right) \) is

\[ t_j^0 = \frac{k \left( e^{(n-1)r} + (n-1)e^{-r} \right)}{\theta \ 2e^{(n-1)r} + (n-1)e^{-r}} \]  

(19)

and the utility is

\[ u_j^0 = \frac{k^2 e^{(n-1)r} \left( e^{(n-1)r} + (n-1)e^{-r} \right)}{\theta \left( 2e^{(n-1)r} + (n-1)e^{-r} \right)^2} \]  

(20)

Eq. (19) and Eq. (20) show how available traffic and related utilities at the Nash equilibrium vary with \( \gamma \). To compare with the classical Nash equilibrium situation and the Pareto optimum situation intuitively, we take the limits for them.

When \( \gamma \to 0 \), we have

\[ \lim_{\gamma \to 0} t_j^0 = \lim_{\gamma \to 0} \frac{k \left( e^{(n-1)r} + (n-1)e^{-r} \right)}{\theta \ 2e^{(n-1)r} + (n-1)e^{-r}} = \frac{nk}{\theta(n+1)} \]  

(21)
When $\gamma \to +\infty$, we have

$$
\lim_{\gamma \to +\infty} Q_j^n = \lim_{\gamma \to +\infty} \frac{k^2 e^{(n-1)\gamma} e^{(n-1)\gamma} + (n-1) e^{-\gamma}}{\theta} = \frac{nk^2}{\theta(n+1)^2}
$$

(22)

Obviously, when $\gamma = 0$, the MNOs game goes back to the original the classical Nash equilibrium situation (see Eq. (4) and (5)) and when $\gamma \to +\infty$, the Pareto optimum situation appears if the they all have the biggest entanglement, falling into the state of $|EPR\rangle$ (Einstein-Podolsky-Rosen, see Eq. (8) and (9)).

If we assume $\theta = 1$, $k = 2$, we can easily observe how available traffic and the utilities at Nash equilibrium vary with reference to the measure of entanglement $\gamma$ and the number of MNOs. (see Figure 2 and Figure 3)

![Figure 2](image2.png)

**Figure 2.** The impact of the optimal available traffic on the entanglement operator $\gamma$ and the number of MNOs.

![Figure 3](image3.png)

**Figure 3.** The impact of the utility on the entanglement operator $\gamma$ and the number of MNOs.

In Figure 2, we show the impact of the optimal available traffic on the entanglement operator $\gamma$ and the number of MNOs. We can see that the equilibrium strategic value of available traffic decreases as the entanglement operator $\gamma$ increases from zero gradually. While the change on the number of MNOs becomes controversial. Obviously when $n \to +\infty$, the value would be 2, that is, the value would tend to 2 with the increase of $n$ if the value of entanglement operator $\gamma$ is sufficiently small. No matter how many MNOs participating in the market, we find that the equilibrium strategic value of available traffic would decrease...
rapidly as long as the increase of γ, even slightly. In Figure 3, we show the impact of the utility on the entanglement operator γ and the number of MNOs. We can see that the equilibrium strategic value of optimal utility increases as the entanglement operator γ increases from zero gradually. Meanwhile, the more value of n leads to the more intense changes of optimal utility with the increase of γ. No matter how many MNOs participating in the market, we find that the equilibrium strategic value of optimal utility would increase rapidly as long as the increase of γ, even slightly. Through the above analysis, we can see the entanglement operator γ stands for the cooperation degree among all the MNOs. If we apply γ in the game reasonably, the dilemma-like situation is thus completely removed.

4. CONCLUSIONS

We investigate the quantization of strategies with continuum space for multiple MNOs in China. We construct a quantum structure where each MNO is given by a qubit and entangles pairwise. Through the research, we find the more value of n leads to the more intense changes of optimal utility with the increase of γ. No matter how many MNOs participating in the market, the equilibrium strategic value of optimal utility would increase rapidly as long as the increase of γ, even slightly. When γ→+∞, the MNOs game goes back to the original the classical Nash equilibrium situation and when γ→0, the Pareto optimum situation appears if they all have the biggest entanglement, falling into the state of |EPR>.

REFERENCES


