Optimized Decision Feedback Equalizer Algorithm based on Sparse Underwater Acoustic Channel

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Abstract

A new optimized blind equalization algorithm for sparse underwater acoustic channel is proposed in this paper. The decision feedback equalizer is a common nonlinear equalizer to overcome the effect of noise in underwater acoustic communication. The decision feedback equalizer algorithm tracks the underwater acoustic channel change and utilizes the feedback information to adjust equalizer. The great calculation amount and complex structure lead the reduction of the algorithm performance. The proposed algorithm reduces the calculation amount with the help of simulated annealing algorithm based on sparse underwater acoustic channel and reaches the effectiveness. Based on the equalization model, various experimental performance analysis has proved that the convergence rate of the proposed algorithm was higher than common decision feedback equalizer algorithm.

Key words: Blind Equalization, Decision Feedback Equalizer, Simulated Annealing Algorithm, Sparse Underwater Acoustic

1. INTRODUCTION

The inter-symbol interference is an important factor of restricting the modern communication quality in communication system. In order to reduce inter-symbol interference, the appropriate compensation is used so as to decrease bit error rate and improve communication quality. The traditional ways to overcome the inter-symbol interference is to add the equalizer on the receiving end which the character of the equalizer is opposite to the character of the channel. However, the channel characteristic of the digital communication system is unknown and time-varying. The adaptive equalizer which can track the channel change and adjust the equalizer weights is born to compensate the channel character accurately. The traditional adaptive equalizers will send the training sequence which is known to the receiving end, the receiving end adjust the parameters of the equalizer according to the error of the training sequence which can lead the equalization finally (Giridhar, Shynk and Iltis, 1992).

The adaptive equalization technique is a hotspot of communication area. Along with the development of the digital communication technique, its deficiency and drawback is obvious which is shown as follows:

1) The communication channel can’t pass information effectively due to the existence of training sequence which occupy large amount of bandwidth and reduce the efficiency of communication system transmission.

2) The communication system will send training sequence frequently because of the equalizer divergency which is influenced by the strong interference of the communication channel and other factors. The frequently sending will disturb the regular work of the communication system.

3) The training sequence is unusable for some specific situation such as information interception and reconnaissance system (Li, 2015; Tugnait, 1996).

To overcome the inter-symbol interference and the shortage of the training sequence, an important way is the blind equalization algorithm. It can save bandwidth and improve the communication efficiency so it has become a hotspot of communication research area. In many blind equalization algorithms, the Constant Modulus Algorithm (CMA) which advanced by Godard and Triechiar is a widely applied method (Godard, 1980; Shalvi and Weinstein, 1990). For quadrature amplitude modulation (QAM) signaling, the MMA2-2 is capable of jointly achieving blind equalization and carrier phase recovery. By selecting appropriate values of p and q, the generic cost function leads to the respective cost functions of several existing blind equalization algorithms (Benvensite and Goursat, 1984; Wesolowski, 1987; Oh and Chin, 1995). A new CM criterion employing fractional lower-order statistics (FLOS) of the equalizer input is proposed in 2004. The associated FLOS-CM blind equalizer, based on a stochastic gradient descent algorithm, is able to mitigate impulsive channel noise while restoring the constant modulus character of the transmitted communication signal (Rupi and Tsakalides, 2004). By generalizing the definition of complex modulus, a family of constant modulus algorithms whose special examples includes not only the well-known constant modulus algorithm, but also the recently
proposed sign Godard algorithm, square contour algorithm (SCA), generalized SCA, and sign SCA is proposed(Gonzalez, Perez and Bravo, 2008). A conjugate gradient and a steepest descent algorithm for real-time processing are proposed and applied to blind adaptive array processor. Multi-user mobile communication systems use adaptive and linearly constrained adaptive filters for blind and non-blind adaptive interference cancelation, multipath reduction, equalization, and adaptive beam-forming (Diene and Bhaya, 2010). Exploiting the error variance relation, the evaluation of excess mean square error (EMSE) of constant modulus equalizers in a noise-free non-stationary environment is discussed. The EMSE analyses are presented for two equalizers - the CMA2-2 and the beta CMA (Abrar, Ali and Zerguine, 2013).

The above algorithms are realized by the linear equalizers. Another family of blind equalization algorithms is nonlinear equalizer blind equalization algorithm. Nonlinear adaptive filters based on a variety of neural network models was used successfully for system identification and noise-cancellation in a wide class of applications. An adaptive Recurrent Neural Network (RNN) based equalizer whose small size and high performance makes it suitable for high-speed communication channel equalization (Keckriotis, Zervas and Manolakos, 1994; Fang and Chow, 1999). Some novel blind equalization approach based on radial basis function (RBF) neural networks are proposed. By exploiting the short-term predictability of the channel input, a RBF neural net is used to predict the inverse filter output. To enhance the identification performance in noisy environments, the improved least square (ILS) method based on the concept of orthogonal distance to reduce the estimation bias. The convergence rate of the ILS learning is analyzed, and the asymptotic mean square error (MSE) of the proposed predictive RBF method is derived theoretically (Xie and Leung, 2005; Raugi and Tucci, 2006; Inan and Erdogan, 2015). A blind adaptation method for the decision feedback equalizer (DFE) is proposed. A DFE is divided into two parts: a front-end linear equalizer (LE), and a prediction error filter (PEF) followed by a feedback part. The weighting coefficients of the filters in each part are updated using constant modulus algorithm and decision feedback prediction algorithm, respectively. The resulting symbol error rate is much less than that of the conventional LE and very close to that of the DFE using a training sequence (Seo, Lee and Woong, 1997; Dogancay, 1998). The Efficient and fully blind Decision Feedback Equalizer (DFE) remains an open issue, mainly because of the potential errors in the decision loop part. Based on the Weighted Decision Feedback Equalizer (WDFE) and other factors, our previous work aiming at decreasing the error propagation phenomena, some new blind DFE which in order to commute also both the algorithms and the filtering structure (Kim, Lee and Kim, 2002; Banovic, Abdel-raheem and Khalid, 2008; Martinez, Menchaca and Berron, 2011).

The study is organized as follows: The system model is introduced in Section 2 especially the difference of linear equalizer and decision feedback equalizer. The optimized algorithm which is utilized with simulated annealing algorithm is discussed in Section 3. The simulation results are presented in Section 4 and the Conclusion is given in Section 5.

2. SYSTEM MODELS

2.1. The Linear Equalizer

Transmission of the training symbols at the beginning end can be impractical for the receiver end initialization. The correct sampling points must be known in extracting the training symbols prior to the convergence of the equalizer. The blind equalization algorithms equalize the channel to correct those demerits without the help of the training sequences and to provide higher data rates. The most widely used algorithm is Bussgang blind equalization algorithm. The transmitted signals are equalized by iteration mode which the data must be nonlinear transformed on the output end of the equalizer. The family of Bussgang blind equalization algorithm has outstanding feature such as simplicity, clearness, low calculation amount and easy realization.

![Figure 1. Simplified communication system.](image)

A simplified modern baseband communication system is described in Figure 1. The signal $x(k)$, assumed i.i.d. (independent and identically distributed) and non-Gaussian is transmitted through an unknown
communication channel. The communication channel is contributed by an FIR filter with its impulse response vector \( h(k) = [h_0, h_1, \ldots, h_{k-1}]^T \) where \((\cdot)^T\) indicates transposition. The white Gaussian noise \( w(k) \) is added in the process of the signal transmission. The input signal of the equalizer is \( y(k) \). \( \hat{x}(k) \) means the output of the equalizer after equalization and the output of the decision device is \( x(k) \). The equalizer weight vector is \( w(k) \).

Considering the aspect of the input signal of the equalizer, there are some fundamental equations can be derived:

\[
y(k) = x(k) * h(k) + w(k) \tag{1}
\]

\[
\hat{x}(k) = y(k) * w(k) \tag{2}
\]

The purpose of the equalizer is to obtain the optimum estimated value of \( x(k) \). Consequently, the equation can be defined:

\[
\hat{x}(k) = x(k - D)e^{j\phi}, w(k) * h(k) = \delta(k - D)e^{j\phi} \tag{3}
\]

where \( D \) denotes a integer delay, \( \delta(\cdot) \) means the the function of Kroneckrt \( \delta \). The Fourier transform of equation (3) is as follows:

\[
W(\omega) \cdot H(\omega) = \int_{-\infty}^{\infty} \delta(k - D)e^{j\phi} e^{-j\omega}dk = e^{j(\phi - \omega)} \tag{4}
\]

Figure 2 is the linear equalizer.

\[
\begin{align*}
\text{y (k)} & \xrightarrow{z^{-1}} \text{y(k-1)} \quad \cdots \quad \xrightarrow{z^{-1}} \text{y(k-L+2)} \quad \cdots \quad \xrightarrow{z^{-1}} \text{y(k-L+1)} \\
\text{w_0 (k)} & \quad \text{w_1 (k)} \quad \cdots \quad \text{w_{L-1} (k)} \quad \text{w_{L-2} (k)} \quad \cdots \quad \text{w_{L-1} (k)} \quad \text{w_{L} (k)} \\
\end{align*}
\]

\[
\hat{x} (k)
\]

\text{Figure 2. The linear equalizer}

The linear equalizer is a usual equalizer in blind equalization. The aim of the equalizer design is to make the system input and the output of the equalizer satisfy the equation (4) by adjusting the tap weighting coefficients according to the blind equalization algorithm. The linear equalizers rely on the constant modulus of the transmitted symbols and the statistical properties of the channel. Most of the communication signals exhibit the Constant Modulus (CM) property and experience loss of this property.

The nonlinear function \( g(\cdot) \) of CMA is as follows:

\[
\hat{x}(k) = g(\hat{x}(k)) = \frac{\hat{x}(k)}{|\hat{x}(k)|} [|\hat{x}(k)| + R_2|\hat{x}(k)| - |\hat{x}(k)|^2] \tag{5}
\]

Here

\[
R_2 = \frac{E[|x(k)|^4]}{E[|x(k)|^2]} \tag{6}
\]

The cost function of CMA is

\[
J(k) = \frac{1}{4}E[|R_2 - |\hat{x}(k)|^2|^2] \tag{7}
\]

The reason for choosing this cost function is that the power of the transmitting signal is constant so the power of the output signal is constant. It can be derived that

\[
e(k) = \hat{x}(k)(R_2 - |\hat{x}(k)|^2) \tag{8}
\]

According to the iteration formula of the steepest descent method, the iteration formula of the weighting coefficients

\[
w(k + 1)w(k) - \mu \nabla(k) = w(k) - \mu \frac{\partial J(k)}{\partial w(k)} \tag{9}
\]

On account of \( \hat{x}(k) = w(k)y(k) \), so

\[
\frac{\partial |\hat{x}(k)|^2}{\partial w(k)} = 2|\hat{x}(k)| \frac{\partial \hat{x}(k)}{\partial w(k)} = 2\hat{x}(k)y(k) \tag{10}
\]

\[
\frac{\partial J(k)}{\partial w(k)} = -E[|R_2 - |\hat{x}(k)|^2| \cdot \hat{x}(k)y(k)] \tag{11}
\]
The CMA expression can be derived when the expected value of stochastic gradient instead of stochastic gradient.

\[ w(k + 1) = w(k) + \mu [R_2 - |\hat{x}(k)|^2] \hat{x}(k) y(k) \]  

(12)

The CMA has fine convergence performance and low computational complexity. It’s easy to realize anyway. The cost function is only relate to the amplitude of the receiving sequence and has nothing to do with the phase. The CMA is the most widely used blind algorithm which restores this property without the source knowledge and can equalize both constant modulus and non-constant modulus (like QAM) signals.

2.2. The Decision feedback equalizer algorithm

The Decision Feedback Equalizer is one of the common nonlinear equalizers. Its basic idea is to estimate and reduce ISI in advance when detecting some bad symbols. The outstanding performance of DFE rely on the equalization between the complexity and performance. It’s constructed by two parts: the first part is feedforward part which are composed with the transversal filters; the second part is feedback part which are consisted of a symbol detector and a feedback transversal filter. The decision device output signal will multiply the tap coefficients and sum together which will send back to the output to overcome the ISI. The structure of feedback has infinite impulse response. So it can compensate the delay distortion of the channel as well as the amplitude distortion. Figure 3 is the structure diagram of DFE:

The tap weighting coefficients of feedforward part are as follows:

\[ w_f(k) = [w_{-N_1}(k), w_{-N_1+1}(k), ..., w_{N_1}(k)]^T \]  

(13)

the length of the feedforward part is \(2N_1+1\). The tap weighting coefficients of feedback part are as follows:

\[ w_f(k) = [b_1(k), b_2(k), ..., w_{N_2}(k)]^T \]  

(14)

the length of the feedback part is \(N_2\). The input recursive vector is \(y(k)\)

\[ y(k) = [y(k + N_1), y(k + N_1 - 1), ..., y(k - N_1)]^T \]  

(15)

The symbol Q means the decision equipment, the vector \(\hat{x}(k)\) is the decision of output signal \(\hat{x}(k)\). The input recursive vector of the feedback part is:

\[ \hat{x}(k) = [\hat{x}(k - 1), \hat{x}(k - 2), ..., \hat{x}(k - N_2)]^T \]  

(16)

Then the output of the decision equalizer is

\[ \hat{x}(k) = w_f^T(k)y(k) + w_f^T(k)\hat{x}(k) \]  

(17)

Here the \(\hat{x}(k)\) is the estimated value of the \(k_{th}\) information code element. It’s clear that the equalizer is nonlinear which the feedback equalizer contained previous code elements. Among the decision feedback equalizer algorithms, the constant modulus algorithm-decision feedback equalizer algorithm is the common blind equalization algorithm which is called CMA-DFE. The cost function of CMA-DFE is

\[ J(k) = \frac{1}{4} E[|R_2 - |\hat{x}(k)|^2|^2] \]  

(18)

The parameter \(R_2\) is a constant:
The iteration formula of the decision feedback equalizer taps are as follows:

\[ w_F(k + 1) = w_F(k) + \mu y(k)e(k) \tag{20} \]
\[ w_B(k + 1) = w_B(k) + \mu e(k)e(k) \tag{21} \]

Here the parameter \( \mu \) is the step size parameter and \( e(k) \) is the error signal of CMA. The CMA-DFE has preferable convergence and robustness which is used widely and favourite for the researchers.

3. THE OPTIMIZED DECISION FEEDBACK EQUALIZER ALGORITHM

3.1. The Simulated Annealing Algorithm

The Simulated Annealing is the method which simulate the annealing of physical material in the industrial disposal. The annealing is a method in industrial disposal. Its inhere manner is to heat up the material to certain temperature and melt it down, then cool it down to the crystal state. In this process, the power in nature in solid can reach the lowest. The annealing is go on step by step when the material is in the steady state.

![Flow chart of SA](image)

The main advantage of SA is: it can discover the best global answer without knowing the derivative of the cost function and is implemented easily. Its basic way is to choose some initialization firstly in which the most important is the length of Markov chain and the cost function; then produce the new data and compare its cost function with the old one; then accept or refuse it in according to certain principle. When the algorithm reach the length of Markov chain, we can adjust the control parameter (the temperature) and go on the next anneal. The common steps of SA are:

1. Set the initial value such as the length of Markov chain, the initial control parameter, the initial model and calculate the cost function of the initial model;
2. Disorder the model and calculate the cost function of the newest;
3. Accept the new model if \( \mu \) or else accept it with the probability of \( P \) which is written in the following manner: in which \( T \) is the control parameter (as the temperature in the annealing);
(4) Under the situation of the control parameter is T, repeat step (2) to step (3) until the method reach the length of Markov chain;

(5) Decrease the control parameter in accordance with the pre-defined way;

(6) Repeat step (2) to step (5) until the method satisfy the stop condition.

Figure 4 is the flow chart of SA:

Simulated annealing algorithm is a well-known optimization method for finding the global optimum, developed by Metropolis et al. The basic idea of the method is to sample the space using a Gaussian distribution. The standard deviation of the sampling distribution is “annealed” with time, as we hone in on the optimum. Simulated annealing was very effective in discovering the global optimum. It is based on seeking the space using a specific fat-tailed distribution. This allows far-reaching access of the state space, and allows much faster annealing and hence faster convergence. Since then, SA has been the simulated annealing method of choice for many global optimization applications.

3.2. The Optimized Decision feedback equalizer algorithm

The aim of the blind equalization algorithm to obtain the optimal tap weighting coefficients by the inverse channel impulse. The start point of blind equalization algorithm and simulated annealing algorithm is alike. A special method to solve the channel equalization is optimization method. Many optimization algorithm have been used in channel equalization(Deng, Li and Xie,2006). The simulated annealing algorithm can also apply to the blind equalization. Here we need a theorem which called Cadzow theorem. We assume sequence \(a_n\) is a stationary random sequence of non-Gaussian and zero-mean; \(s_n\) is the impulse response sequence of association system composed by the non-minimum-phase linear time invariant system and the equalizer; the expected solution of blind equalization is:

\[
\{s_n\} = [\cdots, 0, 0, \beta e^{j\theta}, 0, 0, \cdots] \quad \beta \neq 0
\]  

(22)

In the \(s_n\) sequence, there is the only nonzero element \(\beta e^{j\theta}\) in which \(\beta\) and \(\theta\) respectively stand for the amplitude and phase factor of the ideal system.

For complex signal \(z\), we define the fourth order cumulant and second order cumulant as follows:

\[
\text{CUM}(z: 4) = \text{CUM}(z, z, z^*, z^*) = E[z^4] - 2E[z^2|z|^2] - |E[z^2]|^2
\]  

(23)

\[
\text{CUM}(z: 2) = \text{CUM}(z, z^*) = E[z^2]
\]  

(24)

Here \(\text{CUM}()\) shows the common joint cumulant; * stands for the complex conjugate. If the signal meets \(\text{CUM}(\alpha_n; 4) \neq 0\) and \(\text{CUM}(\alpha_n; 2) \neq 0\), we will define the following normalizing cumulant:

\[
Q(z: 4) = \frac{\text{CUM}(z: 4)}{(\text{CUM}(z: 2))^2}
\]  

(25)

Here \(\text{CUM}(z: 2) \neq 0\). Cumulant shown in equation (2) is identical with the kurtosis in SW criterion, so we can get:

\[
\text{CUM}(y_n: 4) = \text{CUM}(\alpha_n; 4) \sum |s_i|^4
\]  

(26)

\[
\text{CUM}(y_n: 2) = \text{CUM}(\alpha_n; 2) \sum |s_i|^2
\]  

(27)

From \(\text{CUM}(\alpha_n; 2) \neq 0\), it is known that \(\text{CUM}(y_n; 2) \neq 0\). In above equation, \(y_n\) stands for the signal after equilibrium and \(s_i\) is the estimated response. Dividing equation (6) by equation (5), we can get:

\[
\frac{\text{CUM}(y_n; 4)}{(\text{CUM}(y_n; 4))^2} = \frac{\text{CUM}(\alpha_n; 4) \sum |s_i|^4}{(\text{CUM}(\alpha_n; 2))^2 \sum |s_i|^2}
\]  

(28)

\[
|Q(y_n; 4: 2)| = |Q(\alpha_n; 4: 2)| = \frac{\sum |s_i|^4}{(\sum |s_i|^2)^2}
\]  

(29)

It is quite simple for us to prove \(|Q(y_n; 4: 2)| \leq |Q(\alpha_n; 4: 2)|\) (only when \(s_n\) equals \(|Q(y_n; 4: 2)| = |Q(\alpha_n; 4: 2)|\)).

It is obvious that we can finish blind equalization as long as making \(Q(y_n; 4: 2)\) maximized. The necessary condition of the equal sign is to satisfy of a famous theorem that is three is only a zero in the unite channel impulse of the channel and the equalizer. The significance of the Cadzow Theorem is that it give unrestricted condition. Based on the thinking, we can utilize Cadzow Theorem to improve the performance of the equalizer and the new algorithm is achieved. Although it indeed can reflect advantages of high-order cumulant, it is time-consuming and very easy to be converged to local optimal solution. It provides DFE a new way for its simple programming and easiness to understand. The new optimized decision feedback algorithm is called SSADFE whose iteration formula is adopted Cadzow Theorem and utilized the characteristic of sparse underwater acoustic channel. The tap weighting coefficients of the equalizer is sparse for the reverberation structure is
sparse in Ocean acoustic channel especially in high speed underwater acoustic communication with medium and short distance. Sparsity is that most of the energy of the channel coefficient is small, and a few taps are separated from the channel response of the main energy. This phenomenon often occurs in the shallow sea and deep sea convergence zone. Under the sparsity, we can manage it greatly reduce the arithmetic amount and will not influence the total performance.

4. EXPERIMENTAL ANALYSIS AND PERFORMANCE COMPARISON

In this section, we explore the convergence rate and performance of SSADFE through computer simulations. We also compare the performance of the proposed algorithm with the conventional CMA and the CMA-DFE. In the simulations, we have used two complex channel: one is usual underwater acoustic channel which the channel impulse $h_1 = [1, \text{zeros}(1,18), 0.599971]$, the other is a typical sparse underwater acoustic channel which the channel impulse $h_2 = 1 + 0.4z^{-12}$. The channel is normalized such that the total power is unit watts. The equalizer tap number is selected as 11 which the equalizer is decision feedback equalizer the tap number is 22. Additive noise is assumed to be zero-mean complex white Gaussian noise. Transmitted signal length N is chosen as 5000 symbols, which is assumed to be long enough to examine the performance of the three algorithms.

In the first part of the simulation, performance analysis of the three algorithms is performed at 16-QAM modulation. The signal-to-noise ratio values is 15dB. Figure 5 to figure 7 are the constellation diagrams of the three algorithms.

Figure 5. The constellation of CMA

Figure 6. The constellation of CMA-DFE
Figure 7. The constellation of SSADFE

In the first part, figure 5 to figure 7 shows the constellation performance of the three systems at 16-QAM modulation. The figure 7 shows that SSADFE begins outperforming the two other. CMA-DFE performs better than the CMA. CMA seems to have highest SER. But the advantages are not obvious.

Figure 8. The constellation of CMA

Figure 9. The constellation of CMA-DFE
In the second part of the simulations we evaluate the convergence performance of the three systems which the signals are transmitted in typical sparse underwater acoustic. Figure 8 shows the constellations of 5000 elements of the equalizer output at 16-QAM modulation under the given complex channel. From the Figure 8, it can be clearly understood that CMA has lower convergence performance than the two other algorithms. The CMA-DFE has slightly worse convergence than the SSADFE. Figure 10 shows the constellation of 5000 output symbols of equalizer at 16-QAM modulation under spares underwater acoustic channel. From the Figure 10, it can be observed that SSADFE outperforms the two other in convergence rate and convergence performance. CMA-DFE takes the second place and CMA again has the lowest convergence performance.

5. CONCLUSIONS

Despite there is no need for a training sequence, conventional CMA and CMA-DFE has convergence problems. In this paper, we have proposed SSADFE algorithm to eliminate the convergence problem and further improve convergence performance of DFE. We have presented numerical results which indicates the proposed algorithm better resolves the convergence problem. Furthermore, the proposed algorithm has better calculation performance, especially in sparse underwater acoustic channel. SSADFE is a good alternative to the exiting decision feedback equalizer algorithms.

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