Contribution of Rayleigh Damping Parameters to Site Response under Influence of Rayleigh Wave

Shi Youzhi\(^1\,^2\)

\(^1\) School of Naval Architecture, Ocean & Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
\(^2\) School of Civil Engineering and Architecture, Xiamen University of Technology, Xiamen 361021, China

Abstract
To study the surface shallow soil’s Rayleigh wave response characteristics in earthquake, we use the model case in Lamb’s problem to do finite element dynamic analysis, verify the feasibility of the method of finite element simulating the effect of Rayleigh waves. Then, we take Xiamen area’s shallow plain fill and silty clay for our research object, use soil constitutive for the Mohr-Coulomb model, combined with Rayleigh damping, use the method of finite element to study the influence regulation of mass ratio damping coefficient \(\alpha\) and stiffness ratio damping coefficient \(\beta\) in Rayleigh damping on the bottom and boundary of the model with the use of viscous boundary. Studies have shown that finite dynamic analysis can simulate far-field dynamic problems under the Rayleigh wave, and can be consistent with Lamb problem’s analytical solution, but also can catch P-wave and S-wave arrival time; the change of \(\alpha\) makes little difference to Rayleigh wave’s propagation characteristics in plain fill and silty clay; the change of \(\beta\) makes no difference to the propagation speed, but affects wave’s amplitude. \(\beta\) is the master of Rayleigh damping parameters. The conclusion may provide theoretical base and scientific basis for using finite element to study the Rayleigh wave’s response to shallow structure under seismic dynamic wave.

Keywords: Rayleigh Damping, Rayleigh Wave, Surface of Shallow Soil, Seismic Dynamic Response, Finite Element Mechanical Analysis.

1. INTRODUCTION

Compared with the deep underground structure (such as subways, tunnels, etc.), the city’s synthesis pipe gallery or municipal pipeline is generally shallow underground structures. For such structures, surface wave of seismic dynamic propagation should have a significant effect on them. Surface wave is the result of superimposed interference of non-uniform longitudinal and transverse wave and an important wave among seismic wave (He, 1998), and Rayleigh wave is an important part of surface wave. When Rayleigh wave propagates, the particle does elliptical motion backwardly in the plain composed of the direction of propagation of wave and the normal direction of the surface layer, with a large amplitude, and does mostly vertical motion in the ground. Miller (1995) has studied the percentage of three kinds of elastic wave of the total seismic energy input, and found that Rayleigh waves accounted for 67.3%, S-wave accounted for 25.8% and P-wave accounted for 6.9%. Since the energy distribution of Rayleigh wave usually is restricted to the rock-soil layer with a distance of twice as the range of wavelength from half-space free surface, it has a significant influence on the seismic response of the shallow underground structures. Existing earthquake damage shows that underground pipeline suffered serious damage in the earthquake. In 1995 Kobe earthquake in Japan, water, gas and power lines buried underground were all severely damaged. This is mainly due to the longer length of the pipeline and the shallow depth.

The influence of Rayleigh wave may become a controlling factor in structural response. Therefore, studying the field seismic response under the influence of Rayleigh wave has a great significance. Rayleigh wave has now been widely used in geotechnical parameters inversion, geophysical exploration and other aspects, but studies for analysis of Rayleigh wave to field seismic response are still few. Makris (1994, 1995) studied the pile response under the influence of Rayleigh wave; Jiang Dongqi, etc. (2003, 2004) studied the influence of far field vibration on pile response. The above researches both are based on the consideration of Rayleigh wave as harmonic vibration. Yue Qingxia and Li Jie (2008) proposed the concept of approximate Rayleigh seismic wave, using the existing seismic record, viewing it as the fluctuation in the horizontal direction, and then using the Fourier transform to get the vibration in the vertical direction, with the consideration of attenuation in the depth direction, and finally reached the entire displacement field. Luo Tao (2013) gave the entire Rayleigh wave field by using wavelet transform and Fourier transform to the low-frequency reconstruction of seismic wave. The above Rayleigh waves are not constructed by interference waves and have some differences to the actual earthquake vibration. Therefore, exploring the numerical method...
of simulating the influence of Rayleigh wave accurately has a significant meaning on revealing the Rayleigh wave’s influence on shallow underground structures’ earthquake response effect.

Rayleigh wave propagation in the soil is mainly influenced by the geometry damping and material damping of the soil, so that the amplitude and energy of the wave will decline as the distance from focus increases. As the wave spreads around and beneath the soil, the geometric damping has a significant damping effect to fluctuations. This has been taken into consideration in finite element analysis (Wu, 2000). Soil material damping is determined by the friction between the soil particles, the viscosity of water and air voids, which may be considered by using damping model. Among all kinds of damping models, Rayleigh damping model is the most used one (Pan, 2013). Since damping ratio is relevant to frequency in time-domain calculation, damping ratio is mainly controlled by target frequency \( f \). Researchers came up with different methods regarding to the value of Rayleigh damping, also known as target frequency (Duhee, 2004; Hudson, 1994; Annie, 2007; Nozomu, 2002). However, a unified understanding has failed to be formed yet. And the application of it is somewhat inconvenient and needs deeper research.

In this paper, we first established finite element numerical model, analyzed Rayleigh wave propagation velocity and attenuation law, and by comparing to Lamb’s classical analytical solution (Vucetic, 1991) which is obtained by applying a force at the surface of semi-infinite elastic medium to study the propagation of wave, to verify the correctness of numerical model; second, we chose the shallow plain fill and silty clay in Xiamen area as objects and used finite element methods to study the law of the influence of the mass ratio damping coefficient \( \alpha \) and stiffness ratio damping coefficient \( \beta \) in Rayleigh damping on response character; finally, we concluded the seismic wave and Rayleigh wave’s dynamic response character with different soil and different damping and its parameters in Xiamen area, which provided theoretical basis and scientific basis for the influence of Rayleigh wave on shallow underground structure’s seismic dynamic character in Xiamen area.

2. VALIDATION FOR NUMERICAL ANALYSIS OF MODEL

2.1. Classical Lamb’s Problem

In order to ensure the scientificity and validity of the research, we need to first confirm whether the use of the finite element method is accurate enough to simulate Rayleigh waves. This section will apply the finite element dynamic analysis based on the Lamb’s problem model case and compare the result to classical analytical solution to reach the conclusion.

Lamb’s (1904) problem is to study the propagation law of wave in the medium when semi-infinite elastic medium surface is impacted by impact force. Solutions to this problem are many. Among them, Miklowitz (1978), Cagniard (1962), Foinquinos & Roesset (2000) and other people gave the solution to the Lamb’s problem respectively. In this section, by using PLAXIS 2D and PLAXIS 3D program, we constructed two-dimensional axial symmetrical and three-dimensional finite numerical model to simulate Lamb’s far-field problem and compared the finite element calculation result to the closed-form solution given by Foinquinos and Roesset (2000) to verify the availability and validity of the application of PLAXIS dynamic module to solve the far-field propagation problem.

2.2. Model Building

In PLAXIS 2D, we use axial symmetrical model to simulate Lamb’s far-field problem. Take 100m in the model radial direction (x negative direction) and 30m in the model vertical direction (y negative direction) (see Fig.1); in PLAXIS 3D, we build a one quarter model to simulate Lamb’s far-field problem and take 100m in the x negative direction and y negative direction but 30m in the vertical direction (see Fig.2).

![Figure 1. PLAXIS 2D simulating Lamb problem of axial symmetry finite element model](image-url)
We used linear elastic model to simulate site soil, with Young’s modulus of 50000kPa, Poisson’s ratio of 0.25, and unit weight of 20 kN/m³. The axis of symmetry of the model was impacted by a concentrated load as dynamic input. Dynamic load applied a time-varying load which varied according to a triangle form to simulate. The load started from 0.05s and lasted for 0.025s. The load amplitude took 50kN (see Fig.3).

In the finite element dynamic calculation, in order to avoid calculation distortion resulted by the reflection of stress wave at the boundary of the model, it is necessary to ensure the boundary of the model far enough. However, this will increase the number of element and calculation cost. Therefore, it is necessary to introduce the artificial boundary. In this section, we introduced absorbing boundary at the model’s bottom, the boundary of Xmax and of Ymax, to ensure the stress wave to be absorbed at the boundary of model instead of being reflected. The absorbing boundary used by PLAXIS is based on the viscous boundary proposed by Lysmer and Kuhlmeyer (1969).

2.3. Rayleigh Damping

Rayleigh damping matrix C can be represented as the linear combination of the mass matrix M and the stiffness matrix K:

\[
C = \alpha M + \beta K
\]

Where, \(\alpha\) and \(\beta\) represents mass and stiffness proportional damping coefficient. \(\alpha\) decides the influence of mass on system damping. The bigger \(\alpha\) is, the higher the damping of low frequency vibration is; \(\beta\) decides the influence of stiffness on system damping. The bigger \(\beta\) is, the higher the damping of high frequency vibration is. As most of the damping are produced by the propagation of radioactive wave (geometric damping), the Rayleigh damping can be appropriately reduced by using axisymmetric model in the point vibration source’s problem. But for earthquakes and other problems, in order to obtain actual analysis result when using plain strain model, a higher Rayleigh damping is usually preferred.

As we all know, damping would have significant influence on the soil’s response volume and size. Although previous studies have done a lot of work, as for the identification of the damping parameters, an accepted program has still not been reached. Instead, people use some artificial methods to consider material damping and geometry damping. Damping ratio is commonly used engineering parameters. Rayleigh damping coefficient \(\alpha\) and \(\beta\) can be decided by at least two known damping ratio, and these two damping ratio
corresponds to two vibration frequency. If the two frequency of damping ratio are known, it can form two simultaneous equations, thereby solving for $\alpha$ and $\beta$.

In solving the equations of the closed vertical displacement, the introduction of damping parameters can prevent strange numerical oscillation. Therefore, we can also introduce the following Rayleigh damping in the PLAXIS model. Then the study shows that the propagation speed of Rayleigh wave will not be changed greatly when $\alpha_R = 0.001$, $\beta_R = 0.002$.

### 2.4. Simulation Result

Fig.4 shows the curve of vertical displacement and time for the point with a distance of 50m from a dynamic load at the surface of the model, and gives PLAXIS calculation result and Foinquinos & Roesset (2000) analytical solution.

![Figure 4](image)

**Figure 4.** Vertical displacement of PLAXIS simulating results compared to analytical solution at the distance of 50 meters to impact load

It can be said that the displacement amplitude and shape is still consistent despite that the reflection period of pulse is different. Its main difference comes from discrete finite element and time step’s integral. The calculation result obtained from the use of smaller number of element has shown that the applicability of using PLAXIS to simulate far-field dynamic problem. Meantime, the diagram also shows that PLAXIS can capture the arrival time of P wave (compressional wave) and of S wave (shear wave), which cannot be done in Lamb’s equation as it only considers Rayleigh wave.

The propagation characteristics of Rayleigh wave can be observed by transformed grid (2D model) of different time, as shown in Figure 5 (load pulse starts from 0.05s).

![Figure 5](image)

(a) $t = 0.10$ s
Figure 5. Grid morphing shape created by the propagation of Rayleigh wave

As shown in Fig.6, Rayleigh wave’s surface particle track is an ellipse, which is consistent to theory.
3. STUDY OF STRATUM DAMPING PARAMETERS IN XIAMEN AREA

3.1. Parameter Value

Take shallow layer soil in Xiamen area for example, by considering soil property revealed by deep excavation in Hecuo station, Lingdou station, Software Park station and other stations in line 1 of Xiamen subway, we generalized plain fill and silty clay as research objects. Since the measured plain fill and silty clay are at different depth, to make it more comparable, we need to make wave velocity correction by depth.

Adopt following simplified correction formula:

\[
V_{s,2} = V_{s,1} \sqrt{\frac{h_{\text{soil},2}}{h_{\text{soil},1}}} \quad V_{p,2} = V_{p,1} \sqrt{\frac{h_{\text{soil},2}}{h_{\text{soil},1}}}
\]

Where:

- \(V_{s,1}, V_{s,2}\) — the first and second soil layer shear wave velocity;
- \(V_{p,1}, V_{p,2}\) — the first and second soil layer compression wave velocity;
- \(h_{\text{soil},1}, h_{\text{soil},2}\) — the first and second soil layer burial depth.

See Table 1 for the generalization of wave velocity.

<table>
<thead>
<tr>
<th>Station</th>
<th>Burial depth</th>
<th>Plain fill</th>
<th>Burial depth</th>
<th>Silty clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hecuo station</td>
<td>1.8</td>
<td>134</td>
<td>330</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>1.7</td>
<td>153</td>
<td>346</td>
<td>8.2</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>111</td>
<td>265</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>2.3</td>
<td>145</td>
<td>356</td>
<td>5.9</td>
</tr>
<tr>
<td>Lingdou Station</td>
<td>3.5</td>
<td>132</td>
<td>330</td>
<td>13.1</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>135</td>
<td>322</td>
<td>5.9</td>
</tr>
<tr>
<td>Software park station</td>
<td>2.75</td>
<td>135</td>
<td>325</td>
<td>7.36</td>
</tr>
<tr>
<td>Average value</td>
<td>2.75</td>
<td>135</td>
<td>325</td>
<td>7.36</td>
</tr>
<tr>
<td>Generalization value</td>
<td>2.75</td>
<td>135</td>
<td>325</td>
<td>2.75</td>
</tr>
</tbody>
</table>

The value of plain fill parameters is: unit weight \(\gamma=18\text{kN/m}^3\), cohesive force \(c=15\text{kN/m}^2\), frictional angle \(\phi=25^\circ\); the value of silty clay parameters is: unit weight \(\gamma=18.4\text{kN/m}^3\), \(c=37\text{kN/m}^2\), frictional angle \(\phi=15^\circ\).

3.2. Calculation Scheme

We established homogeneous MC material axisymmetric finite element model of plain fill and silty clay respectively, as shown in Fig.7. The material parameters of unit weight, strength and stiffness remain the same, while the value of Rayleigh damping parameters and changes and process the calculation 5 times. The detailed calculation scheme is shown in Table 2, and the corresponding damping curve is shown in Fig.8.

**Figure 7.** Finite element model
### Table 2. Calculation scheme

<table>
<thead>
<tr>
<th>Calculation scheme</th>
<th>Rayleigh damping</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheme 1((\alpha_0\beta_0))</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Scheme 2((\alpha_1\beta_0))</td>
<td></td>
<td>1E-3</td>
<td>0</td>
</tr>
<tr>
<td>Scheme 3((\alpha_2\beta_0))</td>
<td></td>
<td>2E-3</td>
<td>0</td>
</tr>
<tr>
<td>Scheme 4((\alpha_0\beta_1))</td>
<td></td>
<td>0</td>
<td>1E-3</td>
</tr>
<tr>
<td>Scheme 5((\alpha_0\beta_2))</td>
<td></td>
<td>0</td>
<td>2E-3</td>
</tr>
</tbody>
</table>

\[(a) \quad \alpha = 0, \quad \beta = 0\]

\[(b) \quad \alpha = 1E-3, \quad \beta = 0\]
\[
\alpha = 2E^{-3}, \ \beta = 0
\]

(c) \(\alpha = 2E^{-3}, \ \beta = 0\)

\[
\alpha = 0, \ \beta = 1E^{-3}
\]

(d) \(\alpha = 0, \ \beta = 1E^{-3}\)

\[
\alpha = 0, \ \beta = 2E^{-3}
\]

(e) \(\alpha = 0, \ \beta = 2E^{-3}\)

**Figure 8.** Rayleigh damping curve
As can be shown in Fig.8: when $\alpha$ increases, $\beta$ is zero, damping decreases with the increase of frequency; when $\beta$ increases, $\alpha$ is zero, damping increases with the increase of frequency. This indicates that in numerical simulation, $\alpha$ indeed produces significant damping to low-frequency vibration, while $\beta$ produces significant damping to high-frequency vibration. Set five displacement monitor points along the ground surface, with a distance of 40m, 50m, 60m, 70m and 80m from the point of impact load, as shown in Fig.9.

![Figure 9. Displacement monitoring points distribution](image)

### 3.3. Calculation Result

According to the displacement change of previous 5 monitor points at ground surface when model’s axis of symmetry is under the influence of impact load, we drew displacement-time curve with different value of Rayleigh damping parameters respectively. Fig.10 shows vertical displacement response curve of plain fill at site surface.

![Point A (40,0)](image)

![Point B (50,0)](image)
Figure 10. Plain fill site’s surface vertical displacement response curve

We can draw a conclusion from the curve of plain fill site’s surface displacement and time shown in Fig.10 that:
(1) When using MC constitutive model combined with Rayleigh damping parameters to process finite element dynamic time-distance analysis, on condition that Rayleigh damping parameters $\beta$ equals to zero and Rayleigh damping parameters $\alpha$ increases, wave’s amplitude, period and form all remain the same, which means the change of $\alpha$ has no influence on the propagation of wave.

(2) When Rayleigh damping parameters $\alpha$ is zero, by increasing Rayleigh damping parameters $\beta$, the wave’s propagation speed remains the same while the amplitude declines. Therefore, the change of $\beta$ will change the form of wave, which is the value of amplitude. The increase of the value of $\beta$ will decline the amplitude of wave, to stabilize the fluctuation more quickly.

(3) The Rayleigh wave’s frequency of Lamb’s problem in this study is high, $\beta$ is the dominant parameters when using Rayleigh damping.

(4) From Fig.10 (a), we know that Rayleigh wave’s vibration frequency at the response point is $f=1/T = 1/(0.44-0.34) s = 10$Hz, which keeps up well with previous input Rayleigh parameters.

The calculation scheme of silty clay is the same as Table 2. At the horizontal distance of 50m from dynamic load point, the curve of vertical displacement and time under the combination of 5 Rayleigh damping parameters is shown in Fig.11, and we can draw similar conclusion with plain fill.

![Figure 11. Silty clay site’s surface vertical displacement response curve](image)

**4. CONCLUSION**

In this paper, we took the typical site surface soil influenced greatly by Rayleigh wave in Xiamen area as study objects. Where artificial fill is the major study object and shallow burial depth silty clay is the secondary study object, meanwhile using multi-layer media dynamic response to do basic research. Through non-linear finite element numerical simulation, we studied the shallow soil’s dynamic parameters and field’s dynamic response characteristics under the influence of Rayleigh wave, focused on the influence of the change of soil applying different Rayleigh damping parameters on field’s dynamic response and conclusion is as follow:

(1) The dynamic analysis by suing finite element method can stimulate the far-field problem under the influence of Rayleigh wave, and can get an identical solution to Lamb’s problem’s analytical solution. In addition, it can also capture the arrival time of P wave and S wave. Finite element method is useful to the analysis of earth structure’s dynamic response.

(2) The use of MC constitutive model combined with Rayleigh damping parameters to process finite element dynamic time-distance analysis can reflect material damping effect at some extent. Where the change of Rayleigh damping parameters $\alpha$ has little influence on the propagation of Rayleigh wave in typical shallow plain fill and silty clay in Xiamen area; the change of damping parameters $\beta$ does not affect the propagation speed, but wave’s amplitude. Because $\alpha$ determines the influence of mass on the system’s damping, where a greater $\alpha$ leads to a greater low-frequency vibration damping; $\beta$ determines the influence of stiffness on the system’s damping, where a greater $\beta$ leads to a greater high-frequency vibration damping.

The Rayleigh wave’s frequency of Lamb’s problem in this study is high, $\beta$ is the dominant parameters when using Rayleigh damping.

**Acknowledgements**

This paper is also funded by The National Natural Science Foundation of Fujian, number: 2016J01271.
REFERENCE


Luo Tao (2013) “Rayleigh wave research based on wavelet transform and response of the utility tunnel”, *Shandong Jianzhu University, School of Architecture and Construction*.


