Semi-discretization Algorithm for Option Pricing in CEV Jump-diffusion Model

Guojun Yuan

School of Economics and Management, West Anhui University, Lu’an, China, 237012

(E-mail: ygj1010@163.com)

Abstract

This paper proposes an option pricing technique we developed to approximate hedge jump risk under a CEV jump-diffusion model. First, we established the options pricing model and the its partial differential equation by applying the Ito formula and non-arbitrage principle based on approximating hedge jump risk approximation; we next developed the concrete numerical algorithm for the equation by semi-discretizing the spatial variable. Finally, we verified the model’s stability, convergence and effectiveness through numerical experiments on a simulated pricing option scenario.

Keywords: Semi-discretization Algorithm, Option Pricing, CEV Jump-diffusion Process, Stability Analysis, Convergence Analysis

1. INTRODUCTION

An option is a contract which gives its holder the right (but not the obligation) to buy or sell an asset by a certain date for a predetermined price: it can be a “call” option or a “put” option. Options first appeared in the early 1970’s as a finance innovation, then developed quickly into an efficient approach toward risk hedging. In 1973, financiers Fisher Black and Myron Scholes derived the now-famous option pricing model for European plain vanilla (the B-S model), which works by applying a continuous trade strategy based on a few assumptions (e.g., risk neutrality, complete market, the underlying as a geometric Brownian motion) (Black and Scholes, 1973).

Despite the success of the B-S model in the last three decades, there is quite a bit of evidence showing that Brownian motion and normal distribution cannot accurately reflect the market prices of options with various strike prices and maturities. Accordingly, many researchers have attempted to modify the B-S model in order to incorporate two empirical features (Kou, 2003): 1) asymmetrical leptokurticity, that is, where the return distribution is skewed to the left and has a higher peak and two heavier tails than those of normal distribution; and 2) a “smile” in the volatility curve, i.e., a convex curve that accurately reflects strike price. (The B-S model implies that volatility is constant, which in actuality it is not.)

In order to provide the B-S model with the volatility smile feature, Cox and Ross (Cox and Ross, 1976) introduced the constant elasticity of variance (CEV) model, which states the underlying assets of a price process – elasticity of variance, as the name suggests, is a constant in CEV, and asset prices fluctuate according to price levels. Cox (Cheuk and Vorst, 1996) found that the model’s variance smile is caused by negative correlation between the variance and the stock price fluctuations. Davydov and Linetsky (Davydov and Linetsky, 2001) discussed pricing and hedging for path-dependent options, such as barrier and lookback options, under the CEV process; Fusai and Recchioni (Fusai and Recchioni, 2007) analyzed a quadrature method combined with an interpolation procedure for the valuation of discrete barrier options and compared it against GBM, CEV, and variance gamma (VG) stochastic specifications. Massimo (Massimo, 2006) showed how to apply CEV in the case of such options, and developed simple pricing algorithms that compute option price estimates accurately. Hsua, et al. (Hsua, Lin and Lee, 2008) thoroughly reviewed the CEV model and provided detailed derivations. Hoi and Zhao (Hoi and Zhao, 2010) studied American option pricing under the CEV model, and applied the Laplace Carson Transform (LCT) to the corresponding free boundary problem to allow the optimal early exercise boundary determination process to be separated from the valuation process. Sang and Jeong (Sang and Jeong, 2011) derived asymptotic expansions of option prices in the CEV model as parameters which appear in the exponent of the diffusion coefficient corresponding to the B-S model; via perturbation theory, they derived partial differential equations to obtain the relevant results for European vanilla, barrier, and lookback options.

In normal cases, the stock price process is a continuous diffusion process – the CEV process is a continuous time stochastic process, so, accordingly, the underlying asset price is a continuous time function and its expected return will continuously change in a reasonable range. In certain special cases, however, (e.g., natural disasters, coups or other important political events, war, speculative operation) a stock price will undergo large and abnormal motion. The CEV model and its generalized forms can indeed describe the volatility smile of asset prices, but cannot reflect sharp fluctuation-jump phenomena – it is altogether unreasonable to only assume the price process is a CEV process.
The literature contains a number of fairly recent efforts to solve this problem. One notable alternative model to the CEV is the jump-diffusion model, which was first derived by Merton (Merton, 1976): It uses geometric Brownian motion to describe continuous price change and a Poisson jump to describe abnormal price process conditions, and so can accurately describe the price process. After Merton, Bardhan and Chao (Bardhan and Chao, 1995) also studied the jump-diffusion model. Aase (Aase, 1988) developed several important aspects of the modern theory of contingent claims valuation in a frictionless security market with continuous trading, and Amin (Amin, 1993) studied the option pricing problem under a discrete time jump-diffusion model. Scott (Scott, 1997) derived fast closed form solutions for prices on European stock options in a jump-diffusion model with stochastic volatility and stochastic interest rates, and computed the probability functions in the solutions via the Fourier inversion formula for distribution functions. Guokal (Guokal, 2001) derived an analytical valuation of American options based on jump-diffusion processes, and Xu and Jiang (Xu, Qian and Jiang, 2003) conducted a numerical analysis of binomial tree methods for a jump-diffusion model. Wong and Guan (Wong and Guan, 2011) study the option pricing in the leeway process by using the FFT-network. Hilliard and Schwartz (Hilliard and Schwartz, 1996; Hilliard and Schwartz, 2005, Almendral and Oosterlee (Almendral and Oosterlee, 2005), Duan et al. (Duan, Ritchken and Sun, 2006), Feng and Linetsky (Feng and Linetsky, 2008), and Zhang and Wang (Zhang and Wang, 2009) all provide helpful numerical methods for pricing options under stochastic processes with jumps.

Though their contributions were valuable, all the above studies approached option pricing in the jump-diffusion model only based on a density function for the jump term that exists in log-normal distribution; Kou (Kou, 2003) found that for this reason, the existing models do not reflect the reality of the finance market. He developed a new, relatively simple jump-diffusion model in which the logarithm of the asset price is assumed to follow a Brownian motion plus a compound Poisson process, with jumps in double-exponential distribution; in the same study, he attempted to solve the option pricing problem in the double-exponential jump-diffusion model by applying equilibrium theory to derive formules for the standard European option. American option and some path-dependent options. Due to the existence of jumps, the risk neutral probability measure is not unique – Lucas (Robert and Lucas, 1978) and Naik and Lee (Naik and Lee, 1990), for example, attempted to solve the pricing problem in the jump-diffusion process by employing a HARA-type utility function.

Almost any current study on option pricing is conducted based on the assumption that the dynamics of the underlying asset involve geometric Brownian motion, jump-diffusion, or CEV processes. As discussed above, the market prices of options have various strike process and maturities, rendering Brownian motion and normal distribution ineffective – the actual finance market is characterized by asymmetric leptokurticity features and the volatility smile. Merton’s jump-diffusion model and its generalized forms only describe the asymmetric leptokurticity of the return on assets, and the CEV model and its generalized forms only describe the volatility smile of asset prices. These two phenomena exist in the actual finance market simultaneously, however, so in effort to capture them effectively, we established a more general model called the CEV-jump-diffusion model and tested its ability to accurately reflect the underlying assets process; we believe the proposed model can effectively account for the empirical observations found in the literature currently.

Jump-diffusion model techniques applied to pricing options include the Martingale method, stochastic differential equation method, general equilibrium theory method, Merton hedge method, and others. Only the risk related to Brownian motion is hedged in Merton’s method, and Poisson jump risk is substituted by its expectation. Jumping behavior may occur infrequently, but has massive impact on the stocks, bonds and financial derivatives markets when it does – indeed, many stock market crashes in history were caused by sudden, significant stock price volatility. Accordingly, jump risks in the CEV-jump-diffusion option pricing model should be given special attention in terms of the potential impact of jumps on the option price. In this study, we investigated the option pricing problem in the CEV-jump-diffusion model seeking ways to fully hedge jump risk in addition to Brownian motion; the novelty of our approach is that we approximated hedge jump risk by discretizing transaction time, portfolio technology, and the non-arbitrage principle to establish an option pricing model for European call options. We also study the numerical solution for the partial differential equation of the option pricing model.

This paper is organized as follows. The pricing option model in the CEV-jump-diffusion process is derived in Section 2, and Section 3 discusses our concrete semi-discretization algorithm for the option pricing equation derived in Section 2. The convergence and stability of the proposed model are discussed in Section 4, and numerical experiments conducted to verify the model’s effectiveness are shown in Section 5. Section 6 provides a brief summary and conclusion.

2. PRICING OPTION MODEL BASED ON APPROXIMATING HEDGE JUMP RISK IN THE CEV-JUMP-DIFFUSION PROCESS

Let \( S \) be the price process of a risk asset which satisfies the stochastic differential equation in the given probability space \((\Omega, F, P)\):

\[
dS(t) = (r - \frac{\zeta}{2})S(t)dt + \sigma S^\alpha (t) dW(t) + S(t) d\sum_{i=1}^{N(t)} \delta_i
\]

where \( \zeta = E[X_1] \), \( \sigma^2 \) is the variance of the yield of the asset, and \( \sigma > 0 \), \( \sigma \) are constants. \( \alpha \) is also a constant called the elasticity factor, and is the key indicator which determines the relationship between volatility and price.

If \( 0 < \alpha < 1 \), the relationship between stock price and underlying asset price change is positive; the higher the underlying asset price, the greater the stock price change, and vice versa. If \( \alpha < 0 \), the relationship between stock price and underlying
asset price change is negative; the higher the underlying asset price, the smaller the stock price change, (and again, same vice versa,) but the case does not have economic significance because asset price will become negative if the disappearance of fluctuations trends to the starting point. A case of \( \alpha > 1 \) rarely occurs. When \( \alpha = 1 \), the process is a jump-diffusion process.

Here, we assume that \( 0 \leq \alpha \leq 1 \). \( W(t) \) is standard Brownian motion, \( N(t) \) is a Poisson process with rate \( \lambda \), and \( \{X_j\} \) is a sequence of independent, identically distributed (i.i.d.) non-negative random variables from a distribution with the following log-double-exponential density:

\[
f(x) = \begin{cases} p \alpha x^{a_1-1}, & x \geq 1 \\
q \alpha_2 x^{a_2-1}, & x < 1 
\end{cases}
\]

where \( \alpha_i > 1, p, q, \) and \( \alpha_2 \) are positive constants such that \( p+q=1 \).

By discretizing the transaction time during \( \delta \), the underlying asset price \( S(t) \) can be rewritten as follows:

\[
\delta S(t) = (r - \lambda \zeta) S(t) \delta + \alpha S^\alpha (\delta W(t) + \delta \delta t) + \frac{1}{2} \sigma^2 S^{2 \alpha} \delta^2 V(t)
\]

\[
S(t) = (r - \lambda \zeta) S(t) \delta + \alpha S^\alpha \delta W(t) + \Delta S(t)
\]

Let \( V(S(t), t) \) be option price at time \( t \), and the option writer buys \( \Delta \) quantity risk assets at time \( t \), inorder to hedge the option’s risk at maturity \( T \). Cost is \( \Delta S(t) \), so at time \( t \), the payoff is:

\[
\Pi_t = V(S(t), t) - \Delta \cdot S(t)
\]

Then at time \( t + \delta t \), the value becomes:

\[
\Pi_{t+\delta t} = V(S(t + \delta t), t + \delta t) - \Delta \cdot S(t + \delta t)
\]

By applying the Taylor formula (Cont and Tankov, 2004), we have the following:

\[
\delta \Pi = \Delta \delta S + \delta^2 V(t)
\]

The probability is \( \lambda \delta t \) that the Poisson process has one jump during \( \delta t \), and the probability is \( \sigma(\delta t) \) that the Poisson process jumps more than once. So if \( \delta t \) is sufficiently small, (neglecting more jumps,)

then

\[
\delta \left( \sum_{i=1}^{N(t)} X_i \right) = \sum_{i=N(t)+1}^{N(t+\delta t)} X_i \approx \begin{cases} X_i, & P = \lambda \delta t \\
0, & P = 1 - \lambda \delta t \end{cases}
\]

By applying the Taylor formula, we have

\[
V(S(t) + \Delta, t) - V(S(t), t) \approx \delta V + \frac{1}{2} \sigma^2 S^{2 \alpha} \delta^2 V + \frac{1}{2} \sigma^2 S^{2 \alpha} \delta^2 V
\]

If time change \( \delta t \) is sufficiently small, the underlying asset either has no jump or only one jump. We computed an option price when the underlying asset has no jump and has one jump, respectively, where the final option price is the weighted average price.

If there is no jump, the jump-diffusion model is the classic CEV model. Let \( V_1 \) be the option price in this case, then the option price satisfies the following partial differential equation:

\[
\frac{\partial V_1}{\partial t} + rS \frac{\partial V_1}{\partial S} + \frac{1}{2} \sigma^2 S^{2 \alpha} \frac{\partial^2 V_1}{\partial S^2} - rV_1 = 0
\]

If there is only one jump, let \( V_2 \) be the option price and accordingly:

\[
\delta \Pi = \frac{\partial V_2}{\partial t} + (r - \lambda \zeta) S \frac{\partial V_2}{\partial S} - \Delta \delta t + \alpha S^\alpha \frac{\partial^2 V_2}{\partial S^2} \Delta \delta t + \sigma^2 S^{2 \alpha} \frac{\partial^2 V_2}{\partial S^2} \delta^2 t + \frac{1}{2} \sigma^2 S^{2 \alpha} \frac{\partial^2 V_2}{\partial S^2} \delta^2 t
\]

During \( \delta t \) time, the expected return of \( \delta \Pi \) is:

\[
E[\delta \Pi] = \frac{\partial V_2}{\partial t} + (r - \lambda \zeta) S \frac{\partial V_2}{\partial S} - \Delta \delta t + \alpha S^\alpha \frac{\partial^2 V_2}{\partial S^2} \Delta \delta t + \sigma^2 S^{2 \alpha} \frac{\partial^2 V_2}{\partial S^2} \delta^2 t + S \frac{\partial V_2}{\partial S} \xi + \frac{1}{2} \sigma^2 S^{2 \alpha} \frac{\partial^2 V_2}{\partial S^2} \delta^2 t
\]

where \( \xi = Var[X_j] \), and \( E[\delta \delta W(t)] = 0 \), taking \( \Delta = \frac{\partial V_2}{\partial S} + \frac{1}{2} \sigma^2 S^{2 \alpha} \frac{\partial^2 V_2}{\partial S^2} \xi + \frac{1}{2} \sigma^2 S^{2 \alpha} \delta^2 t \), then:
The stochastic volatility and jump disappear, so \( \Pi_1 \) will be risk-free, i.e., \( E[\delta \Pi] = r \Pi_1 \delta t \), then:

\[
\frac{\partial V_2}{\partial t} + rS \frac{\partial V_2}{\partial S} + \left( \frac{1}{2} \sigma^2 S^2 \right) \frac{\partial^2 V_2}{\partial S^2} - rV_2 = 0 \tag{11}
\]

So the option price is 

\[ V = (1 - \lambda \delta t)V_1 + \lambda \delta t V_2. \]

For a European call option, the payoff is 

\[ V(S_T, T) = \max\{S_T - K, 0\}, \]

where \( K \) is the strike price and the behavior on the boundaries is given by 

\[ V(0, t) = 0, \lim_{S \to \infty} V(S, t) = S, t \in [0, T]. \]

For a European put option, the payoff is 

\[ V(S_T, T) = \max\{K - S_T, 0\} \]

and the behavior on the boundaries is given by 

\[ V(0, t) = 0, \lim_{S \to \infty} V(S, t) = 0, t \in [0, T]. \]

3. SEMI-DISCRETIZATION ALGORITHM FOR OPTION-PRICING MODEL

Numerical solutions to a model such as ours must be computed, because no exact solution is available. Typical numerical methods used for this purpose include the finite element approach and finite difference approach; the parabolic partial differential equation is an extension of the ordinary differential equation, in a sense (Iserles, 1996), so the most efficient approach for computing the parabolic partial differential equation is to transform the partial differential equation is into a system of ordinary differential equations. Because discretizing all variables is not particularly efficient, difficulty inherent to applying FD methods for nonlinear models can be overcome by means of linearization strategies that in some way falsify the model, mainly near the maturity and strike price. This issue makes the search for an alternative numerical method which preservesthe advantages of the FD method while allowing appropriate treatment of nonlinearities an especially valuable endeavor. The so-called semi-discretization method (SD) or method of lines (Tavella and Randall, 2000; Smith, 1985; Ballester, Company and Jódar, 2008) applies, here.

Note that Eq. (11) is more complicated than Eq. (7); we focus only on the numerical method for Eq. (11), though the numerical method for Eq. (7) can be derived similarly.

Let \( \tau = T - t \) , \( U(S, \tau) = V_2(S, \tau) \) such that the reversal time question in Eq. (11) becomes the following forward time question:

\[
\frac{\partial U}{\partial \tau} = rS \frac{\partial U}{\partial S} + \left( \frac{1}{2} \sigma^2 S^2 \right) \frac{\partial^2 U}{\partial S^2} - rU \tag{12}
\]

First, replace the partial derivatives by finite difference approximations:

\[
\frac{\partial U}{\partial \tau} \approx \frac{U(S_{i+1}, \tau) - U(S_{i-1}, \tau)}{2h} \]

\[
\frac{\partial^2 U}{\partial S^2} \approx \frac{U(S_{i+1}, \tau) - 2U(S_{i}, \tau) + U(S_{i-1}, \tau)}{h^2} \]  

where \( S_i = K - L + ih, i = 1, 2, \ldots, N - 1 \) are the nodes of the underlying asset interval, \( K \) is the strike price, and \( L \leq K \) is the radius of the neighborhood about \( K \) where the numerical solution is computed. Let \( u_i(\tau) \) be the approximation of theoretical value \( U(S_i, \tau) \), and let \( u(\tau) = [u_1, u_2, \ldots, u_N]^T \). Boundary values \( u_0, u_N \) are computed via the second-order Lagrange interpolating polynomial passing through the two closest internal mesh points, that is: \( u_0 = 2u_1 - u_2, u_N = 2u_{N-1} - u_{N-2} \). The partial differential equation (12) is then approximated by the following system of ordinary differential equations:

\[
\begin{cases}
\frac{du(\tau)}{d\tau} = Cu(\tau), 0 \leq \tau \leq T \\
u(0) = [u_1(0), u_2(0), \ldots, u_N(0)]^T
\end{cases}
\tag{15}
\]

where \( u_i(0) = \max\{S_i - K, 0\}, i = 1, 2, \ldots, N - 1 \).
\[
C = \begin{pmatrix}
    a_1 + 2c_1 & b_1 - c_1 & 0 & 0 & \cdots & 0 \\
    c_2 & a_2 & b_2 & 0 & \cdots & 0 \\
    0 & c_3 & a_3 & b_3 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & \cdots & 0 & c_{N-2} & a_{N-2} & b_{N-2} \\
    0 & \cdots & 0 & c_{N-1} - b_{N-1} & a_{N-1} + 2b_{N-1}
\end{pmatrix}
\]

and
\[
a_i = -\frac{\sigma^2 S_i^{2\alpha} + \lambda S_i + \lambda \xi S_i}{h^2} + rh^2, \quad b_i = \frac{\sigma^2 S_i^{2\alpha} + \lambda S_i + \lambda \xi S_i}{2h^2}, \quad c_i = \frac{\sigma^2 S_i^{2\alpha} + \lambda S_i + \lambda \xi S_i - rhS_i}{2h^2},
\]

\(i = 1, 2, \ldots, N - 1.\)

According to the Euler method, we have
\[
u((n+1)k) = (I + C)\nu(nk), \quad 0 \leq n \leq N - 1 \quad (16)
\]
where \(I\) is \(N - 1\) order unit matrix, \(k = \Delta \tau, \quad mk = \tau.\)

Then the numerical solution to the initial vector problem (16) is as follows:
\[
u(\tau) = \left[ \prod_{k=0}^{m-1} (I + kC) \right] \nu(0) \quad (17)
\]
Further, we can obtain the following finite difference scheme:
\[
F(u^n) = \frac{u^{n+1} - u^n}{k} - rS_i \nabla^n u - \left( \frac{1}{2} \sigma^2 S_i^{2\alpha} + \frac{1}{2} \lambda S_i + \frac{1}{2} \lambda \xi S_i \right) \Delta^n u + ru^n = 0 \quad (18)
\]
where \(u^n = u(S_i, nk)\).

4. STABILITY OF THE SEMI-DISCRETIZATION ALGORITHM

Below, the conditional time stability of the difference scheme is discussed in the sense that a fixed \(h > 0\); that is, when \(h > 0\) is fixed, the solution \(\nu(\tau)\) will remain bounded as \(\Delta \tau \to 0\) or \(M \to \infty\) but with \(M \Delta \tau = \tau.\)

In order to determine the stability of the finite difference scheme, first consider the following definition.

**Definition 1** (Iserles, 1996)

Let \(A = (a_{ij})\) is a \(n \times n\) matrix, then denote \(\|A\|_\infty = \max_{1 \leq m \leq n} \sum_{j=1}^{n} |a_{ij}|\) as the norm of \(A\).

Accordingly:
\[
\|C\|_\infty = \max_{1 \leq i \leq n} \{|a_{i1}| + |b_i| + |c_i|\} = \max(|\sigma^2 S_i^{2\alpha} + \lambda S_i + \lambda \xi S_i - rhS_i| + \frac{\sigma^2 S_i^{2\alpha} + \lambda S_i + \lambda \xi S_i - rhS_i}{2h^2} \leq \frac{2\sigma^2 (K + L)^{2\alpha}}{h^2} + 2\lambda \xi (K + L) + rh(K + L) + rh^2 \leq \frac{M}{h^2}.
\]

So,
\[
\|u(\tau)\| \leq \left(1 + \frac{M}{h^2 \Delta \tau}\right) \|u(0)\| \leq \exp\left(\frac{M}{h^2 \tau}\right) \|u(0)\|
\]
and now the difference scheme is conditionally time-stable as \(h = \Delta S\) is fixed.
5. CONSISTENCY OF THE SEMI-DISCRETIZATION ALGORITHM

The consistency of the difference scheme means that the exact theoretical solution to the partial differential equation approximates well to the exact solution of the difference equation as the step sizes tend to zero (Iserles, 1996). The consistency of the difference scheme is a fundamental property that any effective difference scheme should have.

For $\frac{\partial U}{\partial \tau} - rS \frac{\partial U}{\partial S} - \frac{1}{2} \sigma^2 S^{2\alpha} + \frac{1}{2} \lambda S^2 + \frac{1}{2} \lambda \xi S \frac{\partial^2 U}{\partial S^2} + rU = 0$, denote $L(U) = 0$ and let $F(U^n) = 0$ represent the approximating difference equation defined by (18) with exact solution $u$.

**Definition 2** (Golub and Loan, 1996)

Let $T^n(U) = F(U^n) - L(U^n)$; if when $h = \Delta S \rightarrow 0$, $k = \Delta \tau \rightarrow 0$, then $T^n(U) \rightarrow 0$, the difference scheme is consistent with Eq. (12), where $u^n$ denotes the theoretical solution to Eq. (12) at the point $(S_i, n_k)$, i.e., $u^n = U(S_i, n_k)$.

**Definition 3** (Rafael, Lucas and Jose-Ramon, 2009)

If $T^n(U) = O(h^p) + O(k^q)$, then the difference scheme is consistent with order $(p, q)$.

Considering Eqs. (12) and (18) in terms of internal points:

$$F(U^n) = \frac{U^{n+1} - U^n}{k} - \left(\frac{1}{2} (\sigma^2 S^{2\alpha} \xi \xi S + \lambda S^2 \xi \xi S) \frac{\partial^2 U}{\partial S^2} + rU \right) = 0 \quad (19)$$

Assuming that $U$ admits partial derivatives with respect to $S$ up to order four, and using Taylor expansion at point $(S_i, n_k)$, then:

$$\Delta^n(U) = \frac{\partial^2 U}{\partial S^2} (S_i, n_k) + \frac{h^2}{12} \frac{\partial^4 U}{\partial S^4} (\zeta, n_k) = \frac{\partial^2 U}{\partial S^2} (S_i, n_k) + h^2 M_{i}^{(2)} (1) \quad (20)$$

where $\zeta \in (S_i - h, S_i + h)$, $M_{i}^{(2)} (1) \leq \frac{1}{12} \max \{|U^n(1)|, \max \{ \frac{\partial^4 U}{\partial S^4} (S, n_k) \} |, K - L \leq S \leq K + L \}$.

$$\nabla^n(U) = \frac{\partial U}{\partial S} (S_i, n_k) \frac{h^2}{12} \frac{\partial^4 U}{\partial S^4} (\eta, n_k) - \frac{\partial^4 U}{\partial S^4} (S, n_k) + h^2 M_{i}^{(2)} (2) \quad (21)$$

where $\eta \in (S_{i+1}, S_i)$, $\gamma \in (S_{i-1}, S_i)$, $M_{i}^{(2)} (2) \leq \frac{1}{6} \max \{|U^n(2)|, \max \{|\frac{\partial^4 U}{\partial S^4} (S, n_k) |, K - L \leq S \leq K + L \}|$.

Assuming that $U$ admits partial derivatives with respect to $t$ up to order two, and using Taylor expansion at point $(S_i, n_k)$, then:

$$\frac{U^{n+1} - U^n}{k} = \frac{\partial U}{\partial \tau} (S_i, n_k) + \frac{h^2}{12} \frac{\partial^4 U}{\partial \tau^2} (S, \psi \xi S) = \frac{\partial U}{\partial \tau} (S_i, n_k) + k M_{i}^{(3)} (3) \quad (22)$$

where $\psi \in (n_k, (n_k + 1))$, $M_{i}^{(3)} (3) = \frac{1}{12} \frac{\partial^4 U}{\partial \tau^2} (S, \psi \xi S)$, and $|M_{i}^{(3)} (3)| \leq \frac{1}{2} \max \{|U_i (3)|, \max \{ \frac{\partial^4 U}{\partial \tau^2} (S, \psi \xi S) \} \psi \in (n_k, (n_k+1)) \}$.

So $T^n(U) = F(U^n) - L(U^n) = \frac{h^2}{12} (\sigma^2 S^{2\alpha} \xi \xi S + \lambda S^2 \xi \xi S) M_{i}^{(3)} (3) - h^2 S M_{i}^{(2)} (2) + k M_{i}^{(3)} (3)$.

Thus:

$$|T^n(U)| \leq \frac{h^2}{12} (\sigma^2 (K + L)^{2\alpha} + \lambda S^2 (K + L) + \lambda \xi (K + L)) \max \{|U^n(1)|, 2r(K + L) \max \{|U^n(2)| \} \} + \frac{k}{2} \max \{|U^n(3)| \}$$

where $\max \{|U^n(3)| = \max \{|\frac{\partial^4 U}{\partial S^4} (S, n_k) |, K - L \leq S \leq K + L \}$.

The truncation error of the finite difference scheme (18) is thus bounded and $T^n(U) = O(h^2) + O(k)$, that is, the finite difference scheme (18) is consistent of order (2,1).

6. NUMERICAL EXPERIMENTS

**Example 1**

Consider the convergence and stability for the European call option price in the CEV-jump-diffusion model for different underlying asset current prices with the following parameters: $r = 0.06$, $\sigma = 0.2$, $S_0 = 50$, $K = 50$, $T = 0.5$, $\alpha = 0.6$, $\xi = 0.125$, $\xi = 0.04$.

Table 1. Numerical solution for European call options in CEV-jump-diffusion model.
As shown in Table 1, the algorithm is stable and convergent, and option price tends to a constant when the price step number \( N \) and time step number \( M \) increase; this demonstrates that the method has favorable convergence and can be used during actual options market operation. In said actual application, we can amplify the size of the time and underlying asset price steps to save run-time and obtain higher approximate values.

**Example 2**

Consider the time stability property of the numerical solution with the same parameters as Example 1, taking a fixed value of \( n \), and let \( \Delta u = u(S, \tau, \Delta \tau = k) - u(S, \tau, \Delta \tau = \frac{k}{2}) \).

![Figure 1](image.png)

Figure 1. Time stability of numerical solution.

Figure (1) shows the time stability property of the numerical solution, where the value of \( \Delta u \) decreases quickly when \( M \) increases for different underlying asset current prices.

7. CONCLUSION

This paper presented our study on a novel option pricing method based on approximating hedge jump risk. We first established the option pricing model and its partial differential equation by applying the Itô formula and non-arbitrage principle, then obtained a concrete semi-discretization difference scheme of the partial differential by semi-discretizing the spatial variable. We examined the proposed model’s stability and convergence, then used numerical experiments to show that the algorithm is indeed effective.

ACKNOWLEDGMENTS

This work was supported by the Key Research Foundation for Excellent Youth Talents of the Education Bureau of Anhui Province, China (No. 2013SQRW054 ZD).

REFERENCES


